Using the Rayleigh Model to Assess Future Acquisition Contract Performance and Overall Contract Risk

The future isn't what it used to be. (Yogi Berra)

(Volume I)

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Summary

The holy grail for managers of acquisition contracts would be a method to perfectly forecast cost and schedule for major acquisitions so that there were no surprises as contracts were executed. Unfortunately, we cannot hope to achieve the ability to perfectly forecast outcomes; however, we can hope to achieve an "about right" forecasting tool that

- anticipates cost growth and schedule slips before they happen and
- assesses the risk of their happening at any point in the execution of a contract.

We can also hope for that tool to be able to assess plan risk during source selection and early in contract execution, before actual costs have even been reported.

We believe that the Rayleigh model allows us to construct such a tool. The Rayleigh model is a nonlinear function used to model cumulative cost accrual in development contracts. This model permits us to assess plan realism before a contract starts, and it allows us to assess actual contract execution and overall risk in the contract once work begins.

In this paper, we first rigorously show that our method of estimating the Rayleigh parameters using nonlinear least squares with restricted parameters is the overall best method to efficiently estimate both the cost and the schedule that best fits the data. This method best forecasts cost and schedule outcomes.

We also show how to use this model to approximate the overall cost and schedule risk in a development contract, and how to do this estimation from actual cost data realized during contract execution to date.

We then perform analysis to:

- Show rigorously that the Rayleigh estimating techniques outperform all other widely used estimating techniques for managing contract execution.
- Show in detail how to calculate overall cost and schedule risk derived from actual cost data, which, we believe, will be a new addition to current, widely used methods of contract management.
- Derive "rules of thumb" to guide the general applicability of the Rayleigh model to the management of development contracts.
- Show that the Rayleigh model is a powerful tool for assessing:
  - the realism of contractor offers during source selection;
  - the realism of and the risk associated with a plan for executing a contract, even before actual cost data are available; and
  - the realism of research, development, testing, and evaluation (RDT&E) program funding profiles.
- Develop a robust, user-friendly software application:
  - with a module to assess plan realism or risk; and
  - with a module that includes an expanded set of business insights, to assess contract execution.

We recommend using the Rayleigh model to assess the execution of development contracts and to evaluate those contractual plans for realism even before execution begins. We also recommend the use of our software application to assist the executive in efficiently managing a large and challenging acquisition portfolio.

Introduction

Motivation for this study

In order to report to Navy leadership, senior managers need tools that will give them early warning of cost overrun and schedule slip in major acquisition programs. In this paper, we discuss a model that will aid senior managers.

We note, at the outset, that our suggested solution is not intended as a substitute for detailed cost and schedule analysis, which is entirely appropriate at the program-office level. Also note that our solution does not require vast new or more detailed reporting requirements; a plethora of reports and useful data are already available.

The model we propose uses existing reports to provide analysis and predictions that are "about right." Senior managers can then use this information to oversee and manage their programs with more accuracy than with any other techniques currently in use. Additionally, the solution we propose is more robust, in that it does not depend entirely on the accuracy of earned value management data. Our model mostly depends on actual cumulative cost data, and as a result, it suits our purposes regardless of whether the earned value data are "bad" or not.

The basis of our model is the Rayleigh distribution. We demonstrated in a previous study that this model shows great promise for forecasting final cost and schedule of development contracts [1]. We also showed that this model was useful in both assessing plan validity at the outset of a project, and contract execution once work had begun on a project.

Armed with these promising results, we developed a prototype software application to aid the executive in managing a large development contract. Our model was not a cost estimation model. Rather, it used reported cumulative cost data on a contract and got a best fit to a Rayleigh distribution in order to assess contract execution to date and to assess realistically where we might be headed in terms of cost and schedule. What is more, this assessment included a realistic path forward, against which we can measure future performance. In this sense, the focus of our model was forward-looking.

As a result of this earlier work, we realized the benefit of continuing with this work. We needed to validate more rigorously the model against the entire life of a set of completed research and development (R&D) contracts in our database. Further, we needed to do some comparative statistics to answer further questions. For example, were Rayleigh fits to data different across the services? We also had to incorporate risk in our model. Finally, we needed to improve our software application and make the business insights in that application more robust.

Literature review

Since the 1970s, the military acquisition community has used a method called earned value management (EVM) to collect data on contracts and programs for better management. The contractor EVM systems produce data that an analyst can use for assessing cost and schedule performance at any point during the execution of a contract. With this data, the analyst can compute an estimated cost at completion (EAC) for any particular contract. Analysts' ability to forecast final contract cost and completion time is important for managers making decisions affecting the future of a program or contract.

Yet these techniques have problems. First, the traditional techniques developed to get an EAC and an estimated duration use formulae that have no theoretical basis or any model of how we would expect real contract cumulative costs to accrue over the life of that contract. Second, these techniques rely on extensive gathering of earned value data and the validity of those data.

Recent experience suggests that the data may not be as reliable as we would like. For instance, sometimes data are missing, or there are large gaps in the reporting, or there is circumstantial evidence that unstable budgets for contracts and programs may allow contractors to game the system, especially when their contracts reward "good numbers." At least partly because of these problems, earned value data reports are being used less frequently to manage a program or contract and the reliability of the insights derived from the analysis of these data has been called into question.

However, there is a model of the schedule of cumulative cost accrual over the life of a contract. This model, originally called the "Norden-Rayleigh" model (referred to here simply as Rayleigh) was developed in the 1960s to model the cost growth of software projects [2]. Using earned value data for development programs prior to 1990, later work showed that the Rayleigh model was the best fit for the cumulative cost schedule over the life of completed research and development (R&D) contracts in DoD [3].

Researchers developed a technique of estimating the parameters of the Rayleigh model using a Kalman filter and the Multiple Model Adaptive Estimation method [4]. Finally, researchers examined ten programs consisting of 14 contracts to validate the Rayleigh model in the post-acquisition-reform era and to develop a software tool called the Rayleigh Analyzer© [5]. Nevertheless, while much of this work is used in some fashion in the cost analysis community, it is not currently in wide use in the analysis of earned value data for managing and overseeing contracts.

The reality of acquisition cost overruns and schedule slips, sensationalized spectacularly in the form of Nunn-McCurdy breaches, reinforces the need for analysis and tools that are readily accessible to senior managers in order to exercise their role in oversight of major acquisition programs. However, to be useful, this analysis must be available early in the life of a program so that senior managers can respond to any potential problems in the program and get it back on track.

In addition, it is extremely important to understand that senior managers do not need more reporting requirements, nor do they need more exotic forms of analysis, nor do they need a perfect prediction of final program cost and schedule. What they do need is basic analysis that is relatively easy to implement and does not depend on acquiring any new data. It would be useful if this new analysis technique could be made robust enough to still be "about right" even when data are missing, when there are gaps in reporting of program execution, when data are reported sporadically, or when data submissions show signs that the system is being gamed.

Finally, the model should be better than currently used estimating techniques and lend itself to use by senior managers, not by analysts. The model must address information requirements of senior managers, it must be easy to implement, and it must provide them useful information for making timely and good decisions. It will certainly be true that if senior managers are using the model and earned value data to make decisions, then analysts, contractors, and project managers will use and care about the model and the realism of the data. The Rayleigh model, with modifications from previous work and with the appropriate tools to display the results to decision-makers, is just that model.

Plan of this paper

In this memorandum, we first review the Rayleigh model and our methodology, which establishes how we estimated the parameters of the model. We then show how we are able to use these parameter estimates to estimate final cost and schedule for R&D contracts. Following that, we assess how robust these estimates are. Finally, we calculate estimates of the variance-covariance matrix, which allows us to visualize the risk in our contract estimates.

We address the following research questions:

- Does our method of estimating cost and schedule with the Rayleigh model outperform other models?
- In our previous work, we found that the Rayleigh estimates appeared to be the best early in the life of a program; however, these estimates still underestimated final cost by 30 percent on average. Can we account for this "missing 30 percent?"
- Can we develop broad "rules of thumb" for applying our model to R&D contracts?
- Can the model be used in source selection?

- Can we update and improve our software application tool?

As we answer each of these research questions, we detail our findings. Finally, we offer recommendations for implementation and further study.

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Preliminaries

The model

The most common methods currently used for estimating final contract cost and duration are unsatisfactory in that they do not specifically model the cumulative cost expenditures during the course of a contract. Consequently, there may be better ways to estimate that final cost and duration if an appropriate model can be applied to acquisition category 1(ACAT 1) programs whose performances are of particular interest to senior decision-makers.

The Rayleigh schedule of cost distribution over the life of a contract is such a model and has proven to be a very good fit for complete R&D contracts in the DoD. The logic behind the model is straightforward. It postulates that, for a project where many problems must be solved to produce the end product, the efficient expenditure rate of effort is not constant. This assumption is based on the fact that, in many complex projects, the early effort is primarily expended on problem definition, task development, and identification of subsidiary problems to solve.

In those early stages there is typically only enough work to require a relatively small number of man-hours that may be expended efficiently. As tasks and problems achieve a degree of definition, more people may be brought on to do the work associated with those tasks and problems. Tasks may be done concurrently and integrated with the results of other tasks later in the project. Eventually, the efficient expenditure of work effort peaks, and, as more and more subsidiary tasks are completed, that efficient work expenditure rate begins to decrease as the project nears completion [6 and 7].

The Rayleigh distribution is a cumulative distribution function from probability theory, and it is a special case of the Weibull distribution. Its s-shaped cumulative distribution function and single-peaked and asymmetric probability distribution function mimic nicely this logical pattern of cumulative effort and effort expenditure rate. When this effort is expressed in dollars, we have a model of the cumulative cost path over the course of a contract that makes intuitive sense for the type of work being done.

We use this Rayleigh model to mathematically model the cumulative cost path of actual costs. It is expressed as:

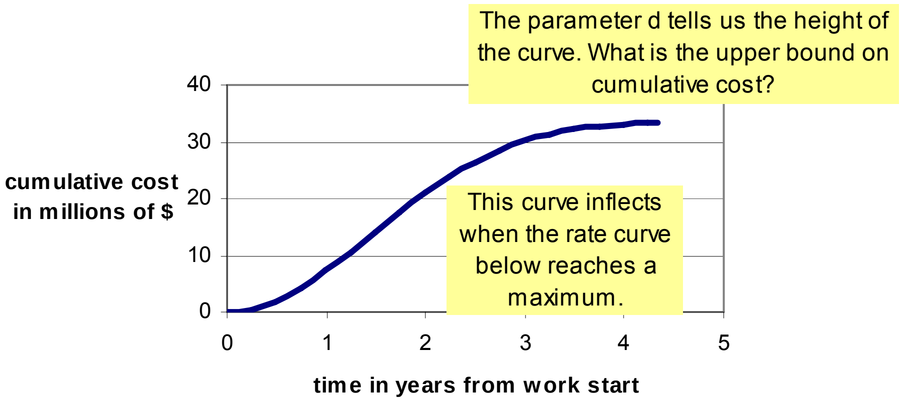
$$c(t) = d(1 - e^{-\alpha t^2}) \tag{1}$$

This mathematical relationship postulates that cumulative cost *c* is a nonlinear function of time, measured in years from the date that work on the R&D contract began. In this model, we observe cumulative cost and time *t*. The scale parameter d is related to the estimated final cost by a mathematical relationship shown in appendix A. The shape parameter α is related mathematically to the estimate of contract duration, also shown in appendix A. The beauty of the Rayleigh model is that we can estimate both final contract cost and schedule by means of these mathematical relationships to the model's two parameters.

The figures below show a Rayleigh curve and its associated expenditure rate function (determined by taking the derivative of the Rayleigh cumulative cost function). These figures illustrate the typical s-shape of the cumulative cost function and the single-peaked, asymmetric shape of the associated expenditure function. The parameter *d* determines the height or the magnitude of the final cost of the project on the cumulative cost curve (figure 1).

The parameter α determines the maximum point on the expenditure rate curve (figure 2). This point, in turn, occurs at the same time as the point of inflexion on the s-shaped cumulative cost curve (figure 1). These figures illustrate graphically how the two functions are related and how the parameters can be interpreted graphically.

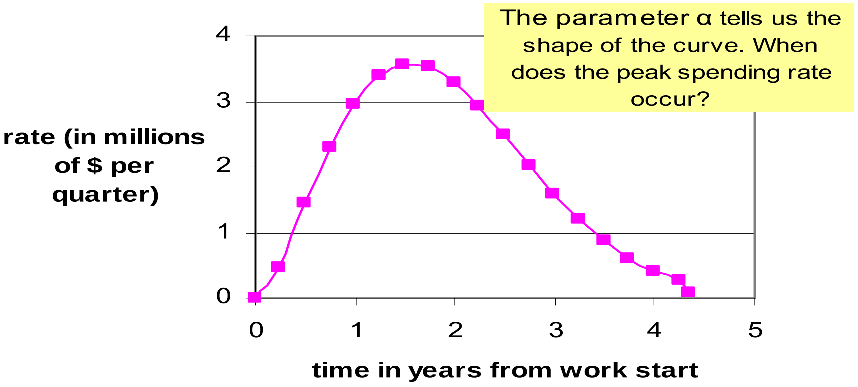
Figure 1. A Rayleigh cumulative cost schedule



The parameter d tells us the height of the curve. What is the upper bound on cumulative cost?

This curve inflects when the rate curve below reaches a maximum.

Figure 2. The corresponding Rayleigh expenditure rate function



The parameter α tells us the shape of the curve. When does the peak spending rate occur?

Since the nature of the Rayleigh function is such that *d* is an asymptotic limit of the cost as *t* approaches infinity, and since no real project will last forever, we must establish a tolerance level within which we will consider the project to be completed, much as we would do in using a numerical method to solve a mathematical equation for which there was no closed-form solution. Other researchers have stipulated that a project is done when the cumulative cost is within three percent of the asymptotic limit *d*. This was done to fit the "rule-of-thumb" that 60 percent of a project's cost will usually be expended at the halfway point of the duration of that project [4].

We will stipulate that a project is done when the cumulative cost is within 1 percent of the asymptotic limit *d*. This will result in the same expenditure accrual at the same point in time as the 3 percent rule.

But, in addition, this 1 percent threshold will accommodate the possibility that a work package on the critical path may occur late in the project and will accommodate the fact that many R&D contracts go well past the 3 percent tolerance level. Mathematically, if we let c_f be the final cost of the project, this final cost will be equal to $0.99d$. This additional relation allows us to derive a well-known relationship between the parameter α and the completion time or duration of the project, t_f (which we show in appendix A). This, in turn, allows us to use statistical estimation techniques to estimate the parameter d and α, and then use those estimates to obtain an EAC using Rayleigh and an estimate of the completion time or duration of the project, also using Rayleigh.

The data

To validate the Rayleigh model we used data from DoD's Contract Analysis System (CAS) database. We selected 74 programs consisting of 107 R&D contracts, which are all completed. The earliest start date for the contracts in our database is 1 January 1970. The latest start date for the contracts in the dataset is 1 August 2002. There were 36 Navy contracts, 36 Air Force contracts, and 35 Army contracts in our dataset.

So that we could assess the quality of forecasts, we needed a datum for comparisons. We used the final reported program manager's (PM's) estimate of final cost as the actual realized final cost. We used the contractor's and the PM's final estimated completion date to derive the contract duration. We treated this contract duration as the actual realized duration. We understand that this is not perfect, but we applied it consistently across all comparisons and datasets. This gave us a basis against which to measure the accuracy of cost and schedule forecasts.

The methodology

We used the common method of least-squares estimation to estimate the Rayleigh parameters. Our basic model is expressed stochastically as:

$$c_i = d(1 - e^{-\alpha t_i^2}) + \varepsilon_i \tag{2}$$

In this model, c_i represents an individual observation of cumulative cost corresponding to an individual observation of time t_i. The error term ε_i is stochastic and is assumed to be distributed normally, with zero mean and a homoscedastic variance ($N(0,\sigma^2)$). We obtain a number of cost and time observations from a particular contract dataset, and we find the value of the parameters d and α that minimizes the sum of squared errors, commonly called the residuals. This estimates the parameters in such a way as to get the closest fit to the data. Additionally, this maximizes the coefficient of determination, which we denote R2. The closer R2 is to 1.0, the better the "goodness of fit," which is to say the better the model explains the variation in the data. The closer R2 is to 0.0, the worse the goodness of fit. This suggests that the model does not explain the observed variation in the data at all.

We applied this model to information sets within each contract data set. We felt that there should be at least three observations of time and cost in order to:

- get reasonable estimates of parameters,
- allow the data to be, in fact, nonlinear, and
- get better estimates of variance.

So for a given contract dataset, our first information set would be the first three observations of cost and time. Using these data, we estimated the parameters. From this we were able to estimate final cost and schedule, based only on the information known at that point in time. We then did the same procedure for the first four observations, then the first five observations, and so forth, up to the full set of all observations.

From this drill, we were able to construct profiles that showed the trends of cost and schedule predictions from very early in the execution of a contract, until the very end of the contract. To do this for all our contract datasets we ran thousands of regressions. Samples of each type of profile are shown below in figures 3 and 4.

In the legend for the **cost prediction profile** graph (figure 3), **EAC_r** represents the Rayleigh model's prediction of final cost at that point in time with only the actual cost of work performed (ACWP) data known at that point. **EAC1**, **EAC2**, and **EAC3** represent the common earned-value techniques for estimating EAC. The variable **ctr_est** is the contractor's estimate of final cost at that point in time. The variable **PM_est** is the PM's estimate of final cost at that point in time. The variable **real_cost** represents the final PM's estimate of final cost and is used as the datum to measure the accuracy.

In the legend for the **duration prediction profile** graph (figure 4), **t_f** represents the Rayleigh estimate of contract duration in years from work start at that point in time. The variable **ctr_t_f** is the contractor and PM's estimate of contract duration at that point in time. The variable **real_dur** is the final contractor's estimate of duration used as the datum.

Figure 3. A sample cost prediction profile

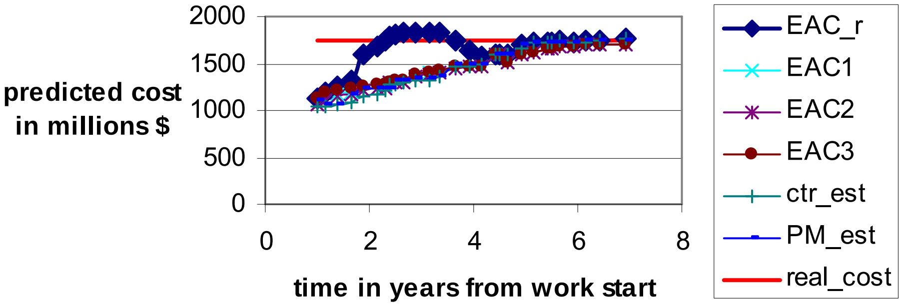
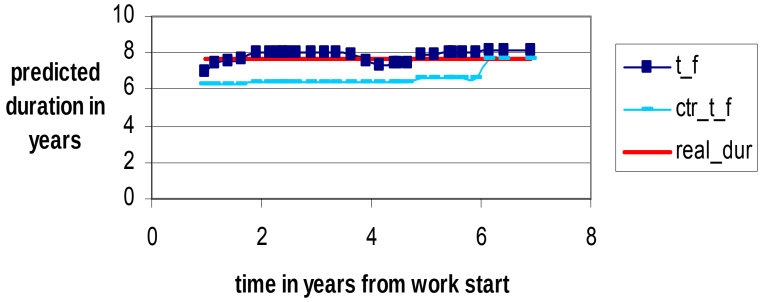


Figure 4. A sample duration prediction profile



At this point, we realized that we needed to do two things with our model. First we needed to validate it against the data to confirm whether actual cost data for R&D contracts eventually follows a Rayleigh functional form, as measured by R2. But more importantly for our primary research task, we needed to assess the quality of cost and schedule forecasting over the life of a contract.

We estimated the parameters of the Rayleigh model in two ways. First we did unconstrained minimization of residuals to get the best estimates of the two parameters of the model shown in equation 2. We called this method nonlinear least squares with unrestricted parameters (NU). This method is a global optimization and clearly yields the best fit to complete contract data. However, the trends of cost and schedule predictions—as evidenced in their corresponding cost and duration profiles over the life of a contract—were erratic, unstable, did not converge, or were generally unreliable.

To alleviate this problem we again minimized the residuals to estimate the two parameters of the model shown in equation 2, but this time we restricted the parameters. This constrained optimization problem had the following constraints: $d_0 \leq d \leq d_{\max}$ and $\alpha_0 \leq \alpha \leq \alpha_{\max}$ [8]. The lower bound d_0 was the maximum of the various EAC estimates divided by 0.99, to include the contractor's and the PM's EACs, or the total allocated budget divided by 0.99, if there was no EAC estimate. The upper bound $d_{\max}$ was simply twice the lower bound. The upper bound $\alpha_{\max}$ was derived from the contractor's and the PM's estimated completion date, or the contract duration. We multiplied this duration by 1.25 to derive the lower bound α_0.

These bounds allowed us to do a grid search for a local optimum to minimize the sum of squared errors. We wrote our own optimization code to do this, and it turned out to be very efficient. The method of computing these bounds is consistent with the stylized facts that R&D contracts inevitably end up costing more and taking longer to complete than initially thought. We called this methodology nonlinear least squares with restricted parameters (NRP) [8].

We compared the resulting fits to complete contract data for all 107 contract datasets using these two methods (NU and NRP). We found that NRP was quite good at fitting complete contract data, although not as good as NU. However, when we compared prediction profiles, we noticed a distinct difference. In 98.0 percent of the cases (105 out of 107), the NRP method generated more reliable and stable cost estimates over the life of the contract. In 100.0 percent of the cases, the NRP method generated more reliable and stable duration estimates.

As a result, we conclude that NRP yields fit to data that are almost as good as the NU method and it yields cost and duration prediction profiles that are far superior. The NRP method yields profiles that are less volatile, that converge faster, that are more accurate and that give fewer false positives. This conclusively demonstrates that NRP is the best method for estimating the underlying parameters of the model. Detailed comparisons are available in the limited distribution volume of this memorandum in (appendix B of Volum II).

A great advantage of using the Rayleigh model is that it allows us to estimate a covariance matrix from which we can infer risk assessments, arising from the data itself [8, 9]. The basic relations for calculating covariance and constructing confidence regions is given by the well-known statistical relations for nonlinear models:

$$Cov = s^2 (V^T V)^{-1} \tag{3}$$

$$(\theta - \hat{\theta})^T V^T V (\theta - \hat{\theta}) \le 2s^2 F(2, n-2; .005) \tag{4}$$

In equation 3, the covariance matrix is a function of s^2, the sample variance; and V, an n by 2 matrix, each of whose elements is the partial derivative of the Rayleigh function with respect to each parameter, evaluated at the estimated value of that parameter and at the particular observed time.

Equation 4 shows the quadratic form of the confidence region where θ is the vector of parameters, $\hat{\theta}$ is the vector of the estimates of those parameters, and $F(2,n-2;.005)$ is the F-statistic with 2 and n-2 degrees of freedom, for a level of confidence of .005 [8]. Equation 4 allows us to graph the confidence region for a set of parameter estimates, given an information set of cost and time data on a contract.

We wanted to develop a metric to represent a confidence region. We calculated this region for a set of data and parameter estimates. We then mapped this approximate confidence region from parameter space into cost-time space, the variables of real interest to the decision-maker. We then assumed that our prior distribution of cost and time duration for a contract was uniformly distributed over a confidence region. With this assumption, we were able to write efficient code to solve the following differential equations:

$$\int_{t_{\min}}^{t} [\rho_u(s)-\rho_l(s)]ds \Big/ \int_{t_{\min}}^{t_{\max}} [\rho_u(s)-\rho_l(s)]ds = p \qquad (5)$$

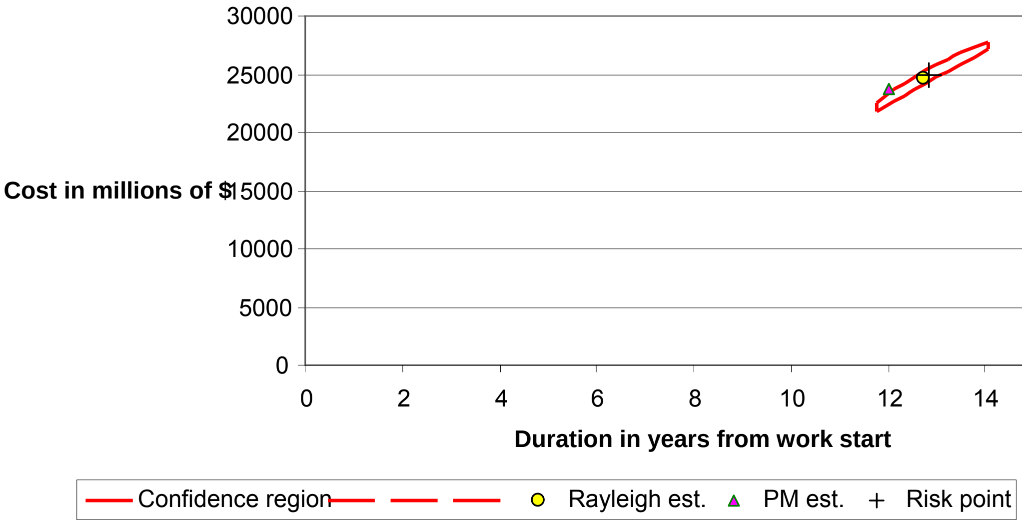
$$\int_{c_{\min}}^{c} [\rho_2(s)-\rho_1(s)]ds \Big/ \int_{c_{\min}}^{c_{\max}} [\rho_2(s)-\rho_1(s)]ds = p \qquad (6)$$

In equation 5, $\rho_u(s)-\rho_l(s)$ represents a vertical slice of the confidence region, $t_{\min}$ represents the minimum time coordinate of the confidence region, and $t_{\max}$ represents the maximum time coordinate of the confidence region.

In equation 6, $\rho_2(s)-\rho_1(s)$ represents a horizontal slice of the confidence region, $c_{\min}$ represents the minimum cost coordinate of the confidence region, and $c_{\max}$ represents the maximum cost coordinate of the confidence region.

In both equations, p represents the chosen level of risk. For example a choice of 0.5 means we want to find the cost or the time in our confidence region where there is approximately a 0.5 probability that cost or time is below that value, and approximately a 0.5 probability that cost or time is above that value, given our previously stated assumption of uniform distribution over the confidence region. We then solve equations 5 and 6 numerically for c and t, the coordinates corresponding to our chosen level of risk. The coordinates are plotted as a risk point. Figure 5, below, is a graphical representation of a typical confidence region.

Figure 5. A typical confidence region with calculated risk point



This methodology provides a powerful tool to visualize and assess cost and schedule risk. This calculation is derived from actual cost data and gives the decision-maker a useful measure of the inherent cost and schedule risk in a contract at a particular point in time.

The methodology has great promise. It clearly has great intuitive appeal for modeling cost accrual over the life of R&D contracts. By its structure with two parameters, the Rayleigh model gives us the opportunity to fit actual cost data to estimate both parameters and estimate both cost and schedule outcomes. The statistical estimate of the parameters also offers us a way of approximating contract risk that is based on actual costs accrued in the contract.

In the next section, we discuss the performance of the Rayleigh model, considering the 107 contract datasets at our disposal.

Results

Currently there are commonly used estimating techniques that use earned-value management data. The relationships are:

$$EAC1 = ACWP + (BAC - BCWP)/CPI \tag{7}$$

$$EAC2 = ACWP + (BAC - BCWP)/(.8CPI + .2SPI) \tag{8}$$

$$EAC3 = ACWP + (BAC - BCWP)/(CPI \cdot SPI) \tag{9}$$

$$CPI = BCWP/ACWP \tag{10}$$

$$SPI = BCWP/BCWS \tag{11}$$

All of these techniques depend on earned value management system (EVMS) data. The primary purpose of EVMS is to plan work by breaking it down into packages, to sequence the work over time, and to manage the execution of that work. A byproduct of this system is a set of techniques for EAC that are indeed quite useful.

In the above equations ACWP represents the actual cost of work performed; BAC represents budget at completion; BCWP represents budgeted cost of work performed; BCWS represents budgeted cost of work scheduled; CPI represents cost performance index, and SPI represents schedule performance index. In addition to the above EACs, we will also compare our method of estimating cost and schedule to the contractor's estimate of EAC, the PM's estimate of EAC, and the contractor's and the PM's estimate of completion date.

Finding 1: Rayleigh (NRP) outperforms other common estimating techniques

Our first goal was to determine whether Rayleigh explained the data over the life of the entire contract. We used the Rayleigh model and the method of NRP to estimate the Rayleigh parameters. From them we were able to infer an estimate of EAC and an estimate of the time duration in years from work start of the RDT&E contract. By calculating the coefficient of determination (the “r-squared” value, which we denote R2), we could measure the fit of our model to the actual cost data for a completed contract. The closer to one that R2 is, the better our model explains the variation in the data, and the better the fit. The opposite is true when R2 is close to zero.

We confirmed that RDT&E contracts will fit a Rayleigh pattern of cost accrual over the life of the contract. We obtained R2 values in excess of 0.9 in 87 percent of the contracts (93 out of 107). We got R2 values between 0.8 and 0.9 in 9 contracts (8 percent) and R2 values between .7 and .8 in 4 contracts (4 percent). In only one case did the R2 value indicate that Rayleigh did not at all explain the data (this contract had an R2 of .046). This demonstrated conclusively that the Rayleigh model as estimated using the NRP technique did a very good job explaining the variation in the actual cost accrual over the entire life of an R&D contract.

We further looked at our database of 107 completed contracts to compare the Rayleigh model across services. There were 36 Navy, 36 Air Force, and 35 Army contracts in our database. The average R2 for Navy contracts was 0.947. The average R2 for Air Force was 0.951. The average R2 for Army contracts was 0.941.

The overall average R2 for all 107 contracts was 0.947. All service averages were remarkably close to the overall average. A standard t-test showed that no service average differed significantly from the overall average of 0.947. This confirms that the ability of the Rayleigh model to explain variation in the data does not differ significantly among the services.

An objection offered by some was that “business practices” have changed since the 1970s so that Rayleigh does not apply any more. To test this idea, we used the year that work started on a contract as an instrumental variable to measure extant “business practices” applying to a particular contract.

We used R2 as a measure of the Rayleigh model’s fit to the data and, hence, its ability to explain variation in the data. We regressed R2 on the work-start year (WSDATE) and a constant. We used 1969 as a base year. So, for example, if a contract started in 1970, its WSDATE Index would be 1. We show this regression and the resulting regression parameter estimates (along with their t-statistics in parentheses) in figure 6 and 7, below.

Figure 6. Test of “evolving business practices”

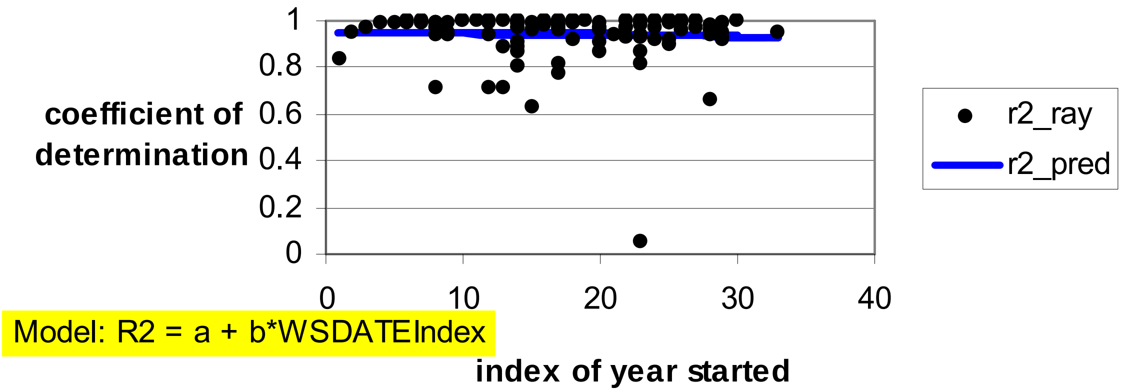


Figure 7. Regression estimates and t-statistics

a_hat	b_hat
0.952	-0.00057
(0.0277)	(0.001443)

If Rayleigh is not applicable any more as “business practices” have evolved, we would expect to see a negative slope to this regression. Instead, we find that the slope term is not significantly different from zero and that the constant term is not significantly different from the overall average R2 of 0.947. This means the functional form of the Rayleigh model explains cost variation as well now as it did in 1970. It may be true that evolved business practices and production methods may reduce the overall cost of contracts over time and compress the duration of contracts over time. However, the functional form that best describes the profile describing the rate at which work will be efficiently accomplished over the life of an R&D contract is still the Rayleigh function.

This observation is made even more concrete by partitioning the set of contracts in our database by the decade. The average R2 for the 1970s was 0.96; for the 1980s it was 0.94; for the 1990's it was 0.93; and for the 2000s it was 0.95. It is apparent that none of these decade average R2s differs significantly from the overall average R2 of 0.947.

In our analysis, we used nominal data. We felt that the effects of inflation would be naturally included already in the cost accrual data in our database. Further we wanted to compare predictions from our estimates of the Rayleigh parameters with predictions of final cost and duration derived by other techniques using earned value management (EVM) data, as well as the contractor predictions and the project manager's (PM) predictions. Because these other methods use nominal data, we wanted to make sure we were comparing "like with like."

To verify that using nominal data should not make a significant difference, we converted cost data and associated EVM data to real data using standard government RDT&E deflators. Examining the resulting Rayleigh fits with real data and comparing our initial results using nominal data revealed that using nominal data produced R2s that were better or almost as good as R2s gotten by using real data 95 percent of the time (102 cases out of 107 contracts).

In fact, the average R2 obtained using our original nominal data was greater (0.947) that the average R2 we got by using real data (0.933). In other words, converting everything to real data made essentially no difference to our fits of the model to data. In fact, conversion to real data made the fits slightly worse on average. This convinced us that using nominal data, that is to say data unadjusted for inflation, was a valid approach.

Having fully examined the ability of the Rayleigh model to fit the data, we next turned to examining the accuracy of the predictions derived from Rayleigh. We compared the full final cost prediction profiles for Rayleigh, EAC1, EAC2, EAC3, the contractor, and the

PM. Figures 3 and 4 show examples of cost and duration prediction profiles for a contract.

We calculated the accuracy measure by integrating the difference between a prediction profile and the eventual "real" outcome over the time that predictions could be made during the life of the contract (from the time of the 3d submission of actual cost data to the time of the last submission of actual cost data). The smaller this measure of total deviation from the profile, the more accurate is the particular prediction profile for cost derived from a given estimation technique under consideration.

To express this more precisely, let dev_{sum} be the sum of the absolute values of the deviations of a particular prediction of final cost from the realized final cost. Let $c_{pred}(t)$ be the prediction profile of all the predicted final costs using any particular method over the time of the contract during which predictions are made. Let c_{real} be the realized value of final cost, let t_3 be the time in years after work start at which the third submission of actual cost data occurs, and let t_n be time in years after work started on the contract at which the last submission of actual cost occurs. Then the measure of total deviation of the cost prediction profile from the realized final cost is:

$$dev_{sum} = \int_{t_3}^{t_n} \left| c_{pred}(t) - c_{real} \right| dt \qquad (12)$$

We evaluated this integral numerically for all types of cost prediction profiles for all 107 contracts. The prediction of final contract cost derived from the Rayleigh model using the NRP technique was the most accurate predictor of final cost 70 percent of the time (75 out of 107 cases). The Rayleigh derived prediction of EAC was best or second best 82 percent of the time.

The EAC1 prediction profile was best 2 percent of the time (2 times out of 107). The EAC2 prediction profile was best 4 percent of the time (4 times out of 107). The EAC3 prediction profile was best 13 percent of the time (14 times out of 107). The contractor's prediction profile was the best 4.5 percent of the time (5 times out of 107). The PM's prediction profile was best 6.5 percent of the time (7 times out of 107). Interestingly, the contractor's or the PM's prediction profile was the worst 51 percent of the time (55 times out of

107). Clearly, on the whole, the Rayleigh model yielded better final cost predictions in terms of relative accuracy.

We used a similar metric to assess schedule forecasting accuracy. Let $tdev_{sum}$ be the sum of the absolute values of the deviations of a particular prediction of final duration from the realized final duration. Let $dur_{pred}(t)$ be the prediction profile of all the predicted final durations using any particular method over the time of the contract during which predictions are made. Let dur_{real} be the realized value of final duration, let t_3 be the time in years after work start at which the third submission of actual cost data occurs, and let t_n be time in years after work started on the contract at which the last submission of actual cost occurs. Then the measure of total deviation of the duration prediction profile from the realized final duration is:

$$tdev_{sum} = \int_{t_3}^{t_n} \left| dur_{pred}(t) - dur_{real} \right| dt \qquad (13)$$

We evaluated this integral numerically for all of the duration prediction profiles for all 107 contracts. The duration prediction profile derived from the Rayleigh (NRP) method was the best over the life of a contract 61 percent of the time (65 out of 107). The contractor's and PM's estimated completion date was the best 35 percent of the time (37 out of 107). The significant effort completion date was the best 4 percent of the time (5 out of 107).

If you just looked at the first three-quarters of the life of a contract: Rayleigh was best at predicting duration 72 percent of the time (77 out of 107); the PM's estimated completion date was the best 26 percent of the time (28 out of 107); and the significant effort completion date was the best 2 percent of the time (2 out of 107). Over the first three-fourths of the life of a contract, the Rayleigh estimate of duration was almost 3 times as likely to be the best predictor of schedule as either of the other two estimated dates. See appendix A, Volume II (in the limited distribution volume) for detailed graphs of fit, cost prediction profile, and duration prediction profile for each of the 107 contracts in our database.

Next, we assessed the usefulness of Rayleigh in providing early warning of cost growth in a contract. Since all techniques yield prediction profiles that converge to the right answer by the end of the contract, we sought to measure which profile converged the fastest.

To measure this we calculated the time in years after work start at which each profile converged to within 10 percent of the eventual right answer. We normalized this time to converge by dividing by the eventual actual duration of the contract. So, for example, suppose we have a contract that takes 10 years to complete. Further, suppose the EAC1 prediction profile converges to within 10 percent of the eventual actual cost of the contract 5 years after work started and subsequently stays within 10 percent of that actual cost. The EAC1 convergence time would, in this case, be .5 (5 divided by 10). The smaller the time to convergence, the better early warning that a particular prediction profile gives. The Rayleigh (NRP) prediction profile converged as fast or faster than any other profile 93 percent of the time (99 times out of 107).

We also wanted to measure when the Rayleigh prediction profile **uniquely** converged faster than all other profiles. This would be a measure of the Rayleigh (NRP) predictor of final cost being a leading indicator of cost growth in a contract. Rayleigh was the **uniquely** fastest to converge to the actual final cost 17 percent of the time (18 times out of 107). The EAC prediction profiles, and the contractor's and the PM's prediction profiles were **never** the uniquely fastest to converge. Hence, good prediction profiles should give early indications of cost growth, and the Rayleigh (NRP) prediction may do this well in advance of any other prediction technique in common use today.

In general, all the EAC estimation techniques were good. However, we found our Rayleigh estimates were superior or at least as good as other EAC estimation techniques. And, as an added bonus, the Rayleigh technique works even when EVM data are missing or nonexistent. For the Rayleigh method to work, all we require are "as of" dates, the date work on the contract started, actual costs, and the total allocated budget for the project. In addition, when Rayleigh diverges from other prediction profiles, this divergence is a good indicator that there is a problem with the EVM reporting system for that project. So the Rayleigh prediction trends can not only diagnose cost and schedule issues before they occur, they may also diagnose problems with EVMS on the particular contract.

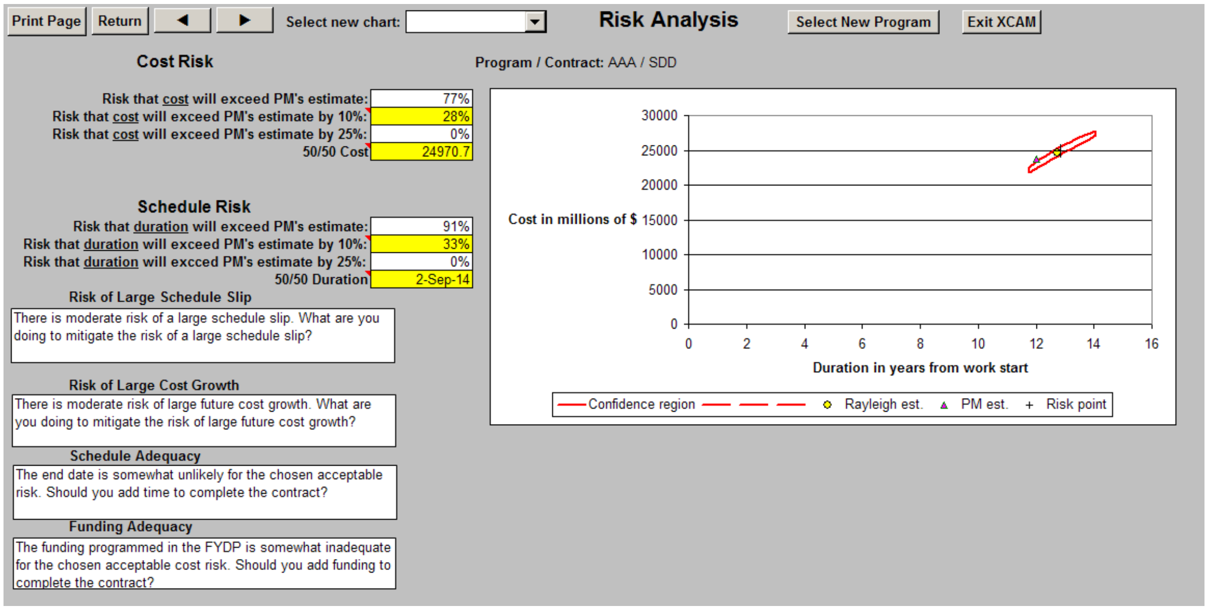
Finding 2: We are able to account for the "missing 30 percent" with risk analysis

As we mentioned earlier, our previous work found that all estimation techniques underestimated final contract cost at the outset of contract execution (measured from the third, fourth, and fifth submission of cost data). Rayleigh, on average, underestimated the least, but still this model underestimated final cost by 30 percent on average. Our more comprehensive analysis for this paper confirms, however, that this "missing 30 percent" is not constant. This is to say that the underestimate may be large early in the life of a contract, but the size of the underestimate diminishes as the contract continues in execution.

This underestimate occurs for two reasons: changing work content and underlying contract risk. To see the first, suppose that an estimate is established for an initial work content in a contract. Now suppose that work content is increased. This will necessarily add time and cost to the contract and will result in higher estimates of cost at completion. In this case, the original estimate was not really low for the actual content of the contract. The original underestimate is only apparent, not real. More generally, though, early in the life of a contract we have less information on its execution and the statistical risk of the estimates is necessarily higher. So much of the initial underestimate is simply a reflection of the inherent contract risk early in the execution of the work under contract.

We attempted to deal with this issue using a spline technique but this was unsatisfactory, because it was really not much more than a curve fitting exercise. Therefore, we concluded from results of risk analysis that the best way to deal with this problem was to calculate the risk region and derive a metric for describing overall contract cost and schedule risk derived from the actual cost data. We described this methodology in the previous chapter. A sample risk analysis for contract data is shown below in figure 8.

Figure 8. A sample risk analysis



Cost Risk

Program / Contract: AAA / SDD

Risk that cost will exceed PM's estimate:	77%
Risk that cost will exceed PM's estimate by 10%:	28%
Risk that cost will exceed PM's estimate by 25%:	0%
50/50 Cost	24970.7

Schedule Risk

Risk that duration will exceed PM's estimate:	91%
Risk that duration will exceed PM's estimate by 10%:	33%
Risk that duration will exceed PM's estimate by 25%:	0%
50/50 Duration	2-Sep-14

Risk of Large Schedule Slip

There is moderate risk of a large schedule slip. What are you doing to mitigate the risk of a large schedule slip?

Risk of Large Cost Growth

There is moderate risk of large future cost growth. What are you doing to mitigate the risk of large future cost growth?

Schedule Adequacy

The end date is somewhat unlikely for the chosen acceptable risk. Should you add time to complete the contract?

Funding Adequacy

The funding programmed in the FYDP is somewhat inadequate for the chosen acceptable cost risk. Should you add funding to complete the contract?

Using our methodology, we can calculate overall cost and schedule growth risk at any point after the submission of three sets of actual cost data. For example, we can calculate the risk that the PM's EAC will be exceeded by at least 10 percent by numerical integration techniques (28 percent in the example in figure 8). We can also calculate the aggregate risk that the contract will go at least 10 percent beyond the PM's estimated completion date by numerical integration techniques (33 percent in the example in figure 8). This gives the decision-maker a good sense of the overall risk of cost and schedule growth based on the current cumulative status of contract execution.

We calculated confidence regions for all 107 contracts in our dataset and took "snapshots" at the quarter, halfway, and three-quarter point in the execution of each contract. This resulted in 428 snapshots.

The calculated confidence region contained the final outcome 78 percent of the time (332 times out of 428 snapshots). The confidence regions consistently got smaller over the life of the contract 89 percent of the time (95 out of 107 contracts). The "center-mass" or 50-percent risk point was a good estimate of final cost 75 percent of the time (319 out of 428 snapshots). The center-mass was a good estimate of the final duration of the contract 61 percent of the time (263 out of 428 snapshots). In this way, we concluded that the risk method with its associated risk calculations was indeed a powerful addition to the analysis. To our knowledge, current analytical techniques in wide use for program management do not assess overall cost and schedule risk in a program. What risk analysis is done is often quite subjective, and it is often related to technical risk.

Even Monte Carlo techniques rely ultimately on subjective risk analysis. Since our technique is ultimately based on common and sparse statistical assumptions and on actual cost data, we conclude that this technique is a superior way of assessing overall cost and schedule risk in a program based on how it is actually executing to date.

Finding 3: We developed "rules of thumb" for the applicability of the Rayleigh model

Our analysis quickly showed that we needed to evaluate and compare our Rayleigh method to other estimating techniques in several dimensions. We settled on five relevant dimensions. They were:

- how well Rayleigh fits the actual data from a completed contract;
- how accurate is Rayleigh relative to other techniques in estimating final cost;
- how quickly does the Rayleigh estimate converge to the final cost in absolute terms;
- how quickly does the Rayleigh estimate converge to the final cost in relative terms; and
- how often is the Rayleigh estimate a leading indicator of cost growth.

We have already discussed how we measured these dimensions.

We calculated the scores for our Rayleigh model for each of the five dimensions and added them to get a cumulative Rayleigh model score for each of the 107 contract datasets in our database. As a check, we visually looked at the graphs of all profiles to confirm the accuracy of our scoring, which was done from the data. The average cumulative score for the Rayleigh model was 22.5. A score above 26 was exceptionally good and a score above 19 was good. A score below 16 was bad.

We found that 97 out of 107 Rayleigh profiles received fair to excellent scores (91 percent). Only 10 profiles (9 percent) received bad scores. This means that when considering all the dimensions of a predictive model, the Rayleigh (NRP) model does exceptionally well in fitting data, predicting cost and schedule, and providing early warning of cost and schedule growth. By examining these scores, and taking a particularly hard look at the bad scores, we developed some rules of thumb and cautions for the use of the Rayleigh model with R&D contracts.

General observations

First, the Rayleigh model only reliably applies to R&D-type contracts. Given this broad caveat, a further general observation is that Rayleigh (NRP) predictions assume that all work for a contract is known at the time the prediction is made. If Rayleigh prediction profiles appeared not to give very good early predictions, we looked at the trends for the contract budget base (CBB), which is a measure of the contract value at the time of a prediction and the trend in the total allocated budget (TAB) for the contract.

Sudden steps upward in CBB and TAB indicate work content has been added to the original contract. Note that we should expect CBB and TAB to be equal. When we see CBB and TAB trends separate with TAB suddenly exceeding CBB, we may properly infer an over-target-baseline (OTB) has been approved and it is possible a replanning of remaining work has occurred. An OTB is a management construct that, with the customer's acknowledgement and without changing contract terms and conditions, permits a contractor to manage and report against a value that is higher than the contract's estimated or target value. See appendix B for a discussion of contracting for EVM.

As an example, in one of our contract datasets, the original predictions by Rayleigh (NRP) for both cost and schedule appear to be well off the final realized cost and schedule. And, neither of these predictions converge to the final realized cost and schedule very quickly. However, an examination of the CBB/TAB trends and a large subsequent schedule slip indicates the addition of work content. The separation of the CBB and the TAB trends indicate a replan as a result of an OTB. Given these two sudden changes, it is hardly surprising that original Rayleigh (NRP) predictions should be so far off (although they are no worse in the case of cost estimation and better in the case of schedule estimation than other methods of cost and schedule estimation).

Once the full content of the work is reflected in the contract and properly planned and priced, the Rayleigh predictions are very close to the final realized values. The original Rayleigh predictions appear to be perfectly correct for the value of the work contained in the contract when the prediction was made.

The opposite case of reduction in work content also occurred in at least three cases among our contract datasets. Again in these cases, the Rayleigh predictions appear to be correct for the value of the work contained in the contract at the time the predictions were made.

This raises a question: Why does the Rayleigh model appear to be a unique leading indicator of eventual cost growth in some cases (for example, figure 3 above), while in other cases Rayleigh does no better than other estimation techniques at anticipating cost growth (as in all of the examples of the previous two paragraphs)? We believe it has to do with the question of whether new work has been added to the original contract or whether existing work turned out to be more costly or time-consuming or both than originally thought.

If there is no change in work content, but the work eventually costs more and takes longer than we expected at the beginning, then we should see a pattern like that shown in figure 3. In this case the overall contract scope has not changed, and the real costs of doing the work are reflected in the actual cost data. The Rayleigh (NRP) model correctly anticipates the cost growth, because this cost growth has been incrementally reflected in actual cost data up to the time the prediction is made. This is a typical case where Rayleigh may be a unique leading indicator of what the eventual realized cost and duration of the contract will be.

Another possible explanation for Rayleigh correctly being a lead indicator of cost growth is that the EVM reporting system may be suspect. Since Rayleigh (NRP) does not rely solely on EVM data reliability to make cost and schedule predictions, separation of the Rayleigh prediction profiles from the other prediction profiles could indicate a problem in the reliability of the EVM system.

If we have no reason to suspect the reliability of the EVM reporting system, it may also be that Rayleigh is "seeing" the cost growth in the reported actual costs to date and revising its EAC estimate upward immediately. The other EAC methods depend on the current TAB for their calculations, so they will not "see" the cost growth until the TAB changes. If these changes do not occur immediately, they may lag behind the Rayleigh estimates in converging to the realized final cost.

If there is a major change in the work content or a major replan of the remaining work, the actual cost data to date will not reflect cost growth (or major cost reduction) until that new work has begun or work under the replan has begun. As a result there is no reason to expect Rayleigh (NRP) to anticipate cost growth (or reduction) faster than other estimates under these circumstances.

Another general observation is that the EAC estimates from Rayleigh (NRP) often mimic the estimates using other currently standard techniques. This may be interpreted to mean that the EVM reporting system is working properly and the data are reliable.

In addition to confirming the quality of the extant EVM system for the contract, Rayleigh (NRP) also predicts contract duration. Further, since it is parametric, Rayleigh (NRP) permits the calculation of variance/covariance matrices and the approximation of overall contract cost and schedule risk based on reported actual cost data alone. None of the currently widely used estimation techniques provide an independent estimate of schedule nor can they estimate cost and schedule risk from actual cost data alone.

A final general observation concerns the estimate of duration. The Rayleigh (NRP) estimate of duration is constrained to be at least as big as the contractor's and the PM's estimate of duration and no more than 25 percent in excess of that estimate. This, along with constraints on the range of final cost estimates, guarantees stable, converging, and less volatile Rayleigh estimates of final cost and duration. Further, we stipulated that the contract ended when the final cost was within 1 percent of the estimation of the Rayleigh model's scale parameter, d. This is a very precise definition of how we calculate contract duration estimates.

Unfortunately, our data are not that precise. Data reporting for our datasets can stop as early as when 90 percent of the work is completed. In this case, Rayleigh must extrapolate the remaining 10 percent of the work to estimate the final cost and duration. In the contracts in our data set, the percent of work complete when the final report was logged ranged from 90 percent to 100 percent, with the average being 95 percent. Thus in each contract, up to 10 percent of the work remained to be completed after the last reported data point. Further, it is unknown what criteria the contractor or the PM used on a particular contract to estimate when work would end under a given contract. Hence, its is not very clear exactly when work on a particular contract stopped because of these reporting gaps and definitional ambiguities.

As a result, we observe that, toward the end of many predicted duration profiles, the Rayleigh estimate of ultimate contract duration may exceed the "real" value of contract duration, which is only the value of the contractor's and the PM's last reported estimate of completion date. We do not know if this standard, against which we measure the accuracy of the Rayleigh duration prediction, is itself accurate. Frankly, we have more confidence in the Rayleigh estimate of completion, as we have defined the criterion for judging when work is completed on a contract.

In any event, Rayleigh provides us with a reasonable independent estimate of contract duration, and moreover, Rayleigh (NRP) permits us to approximate schedule risk. In addition, we note that this anomaly in Rayleigh predicted duration only appears to arise near the end of reporting on a contract in any event. At this point all estimates are converging to the realized final values since the contract is nearing completion anyway. Ultimately, the real added value of the Rayleigh (NRP) technique is its ability to provide early useful predictions and risk assessments to aid management in contract execution before the contract is essentially over.

Specific "rules of thumb" when using Rayleigh (NRP)

After scoring all the contracts in our dataset, we found 10 contracts (9 percent) had low cumulative scores. There were three additional contracts that had fair cumulative scores but relatively poor fits (R2 greater than or equal to .7 but less than .8). A closer examination of this small set of contracts revealed some intuitions about rules of thumb to use when applying the Rayleigh (NRP) model to contract management.

First, we noticed that three of these low-scoring contracts were engine contracts, for which the fits of the Rayleigh (NRP) to the data were satisfactory and the time predictions were satisfactory. However, the cost prediction profiles were poor. The Rayleigh (NRP) generated final cost predictions that were way too high before eventually converging to the final realized outcome well into the execution of the contract and well after the other estimates had converged. We found one exception to this pattern: the engine contract for the V-22 Osprey.

We believe this pattern is observed because engine development for aircraft hardly ever starts from scratch (the Osprey engine may be the exception). Hence, the front-loaded nature of early actual cost data spoofs the Rayleigh (NRP) algorithm into forecasting an unrealistically high prediction of final cost. Because of this, when looking at engine contracts, managers should assess how much new development is actually contemplated being done on the new engine. If the answer is "not much," then Rayleigh (NRP) predictions of final cost early in the program should be disregarded in favor of those generated by other commonly used estimating techniques. If on the other hand, quite a lot of new development and problem-solving is required in developing the new engine, Rayleigh (NRP) predictions of final cost may still be used, and their concomitant risk calculations should still be valid.

There were five contracts where the Rayleigh (NRP) fit to the data was not relatively good (R2 less than .8). From these cases, we observe that, in general, when we get a relatively bad fit to data early in the program and the contract cost accrual appears to be backloaded, an analyst should question the end dates projected by the contractor and the PM.

When Rayleigh (NRP) fits are bad and concern spacecraft or lead ships, a more specific observation applies. Because of the nature of these contracts, the prototype is actually launched and used operationally. As a result, research and development activities and production activities occur in the same contract. So if the fit is bad and appears backloaded, ask whether the contract is for a lead spacecraft or ship. If the answer is yes, then question the end date. In these cases the launch date may be a more appropriate date to use to begin to estimate R&D contract duration and cost for the development portion of the contract only. Another observation is that in these "mixed breed" contracts that contain both R&D activities and production activities, the more appropriate model of cost accrual for the entire life of the contract may be "level-of-effort."

When the Rayleigh (NRP) fit is bad and the costs appear backloaded, it may also be the case that the contract is incomplete. That is to say, some of the development work is being rolled over into the follow-on production contract. Another possibility is that the contract is subject to a cost cap and the contract ran out of money. In either case, these are symptoms of ill-defined, ill-conceived contracts and poor contract execution and management.

Another specific observation is that the goodness of a Rayleigh (NRP) fit to the data may depend on the experience of the contractor. If the fit is poor and the contract is a fixed-price type, it may be plausibly explained by the fact that the contractor is smaller and inexperienced. Such a contractor may not ask for budget adjustments even as costs mount. The contractor may treat the fixed price as a cost cap. In this case, the poor fit of the Rayleigh (NRP) to the data may be indicative of an inexperienced contractor, a high-risk development, or both.

In one example of a bad Rayleigh fit, the contract had a firm fixed price (FFP) component. The contract line item numbers (CLINs) that were FFP were not required to be reported. It is possible that the poor fit of the Rayleigh (NRP) to the reported cost data is because a lot of the costs were not reported because they were not required to be reported.

The Rayleigh (NRP) model was only a fair fit for the data, and the associated final cost prediction profile was terrible in the case of another contract. The costs for this contract appeared to be extremely front-loaded. As a general rule, if the Rayleigh (NRP) fit is fair or worse (R2 less than .9) and the costs are frontloaded, the manager should question the end date, since it should probably be sooner. If the end date appears to be good, then question the Rayleigh (NRP) estimate of the final cost and use one of the other estimates of final cost instead.

In the case of another contract, the Rayleigh (NRP) fit was good but the prediction profile was terrible for the last half of the program. In a case such as this, the culprit may be a cost cap. The contract was on a path to a higher final cost but a cost cap prevented that high final cost from being realized.

Finally, three contracts had Rayleigh (NRP) fits that were excellent, but the prediction profiles were terrible. The early predictions were, in all three cases, way too high. This is an anomalous result and may indicate there were EVM data problems. In cases such as these, the manager should ensure the EVM system for the contract is correct. If it is, then the manager should question the Rayleigh-generated EAC and use an EAC generated by one of the standard estimation techniques instead.

The remaining 94 (88 percent) Rayleigh (NRP) cumulative scores were good or excellent. In all these cases, Rayleigh (NRP) fit the data extremely well and produced predictions profiles for both cost and schedule that were very good, subject to the general caveats on variable work content and OTBs already mentioned.

We were also asked to assess our Rayleigh model's performance when explaining program RDT&E funding. We examined 39 programs, of which 16 were Navy, 9 were Army, 13 were Air Force, and 1 was DoD.

We found there was a high overall goodness of fit (average R2 equal to 0.979). The Navy's average R2 was 0.978; the Army's was 0.987; the Air Force's was 0.976; and DoD's was 0.98. No service average differed significantly from the overall average. We concluded, that we could use the Rayleigh model to assess program RDT&E funding plans. See appendix C, Volume II (the limited distribution volume) for detailed graphs.

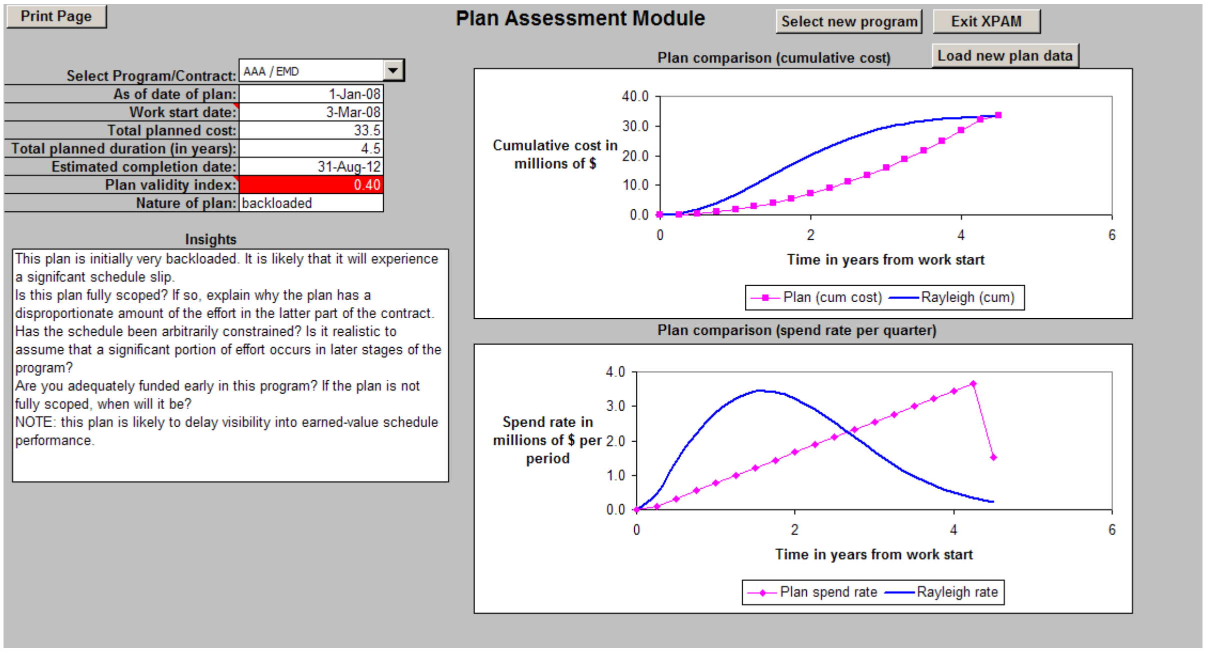
A summary of the "rules of thumb" are:

- Do not use Rayleigh for procurement or "procurement-like" contracts
- Apply Rayleigh with caution to engine contracts
- Apply Rayleigh with caution to contracts with cost caps
- Take care estimating contract duration when the prototype will be deployed and operated (for example, first spacecraft, lead ship, etc.)
- Take care using the Rayleigh estimate of contract duration during the final quarter of a contract
- Use the Rayleigh model to assess program RDT&E funding plans

Finding 4: We used the Rayleigh model to assess plan realism during source selection

If we have a plan and we can reasonably assume the Rayleigh model for the rate at which work can efficiently be done is true for a particular project, we can apply the Rayleigh model to assess the realism of the plan. The Rayleigh model is not a cost estimation model. But, given a final project cost, a date on which work will start, and a date on which work will finish, we can estimate a unique Rayleigh map of the rate at which work can efficiently be done, with a consequent cumulative cost curve for the project cost. Also given a plan for the rate at which work will be accomplished in terms of budget dollars, we can assess the realism of that plan against that unique Rayleigh schedule. Such a sample analysis is shown in figure 9 below.

Figure 9. A sample plan analysis



Select Program/Contract:	AAA / EMD
As of date of plan:	1-Jan-08
Work start date:	3-Mar-08
Total planned cost:	33.5
Total planned duration (in years):	4.5
Estimated completion date:	31-Aug-12
Plan validity index:	0.40
Nature of plan:	backloaded

Insights

This plan is initially very backloaded. It is likely that it will experience a signifcant schedule slip.
Is this plan fully scoped? If so, explain why the plan has a disproportionate amount of the effort in the latter part of the contract.
Has the schedule been arbitrarily constrained? Is it realistic to assume that a significant portion of effort occurs in later stages of the program?
Are you adequately funded early in this program? If the plan is not fully scoped, when will it be?
NOTE: this plan is likely to delay visibility into earned-value schedule performance.

We developed a metric called the Plan Validity Index (PVI) which measures the degree of difference between the plan and a Rayleigh schedule fitting the same final cost and schedule. The relationship showing the calculation of PVI is:

$$PVI = \int_0^T \left|[P(t) - R(t)]\right| dt \Big/ \int_0^T R(t)dt \qquad (14)$$

In this relationship, $P(t)$ represents the cumulative value of the work under the plan over time. The function $R(t)$ represents the cumulative value of the Rayleigh schedule over time. The contract duration is represented by T, measured in years. The PVI is calculated by numerical integration. It is simply the area between the planned cumulative cost and the unique Rayleigh schedule for a given contract value and schedule, normalized by dividing by the area under the Rayleigh schedule.

A PVI of zero would indicate a perfect alignment between the plan and the Rayleigh schedule. A PVI of one would indicate an extremely high degree of difference between the plan and the associated Rayleigh schedule.

You can think of the PVI as a measure of the plan's realism. As such, it is a metric for the amount of risk in the plan. If the PVI is low, this may be a realistic plan and, hence, a less risky plan. If the PVI is high, this is an unrealistic plan, and hence, a high risk one.

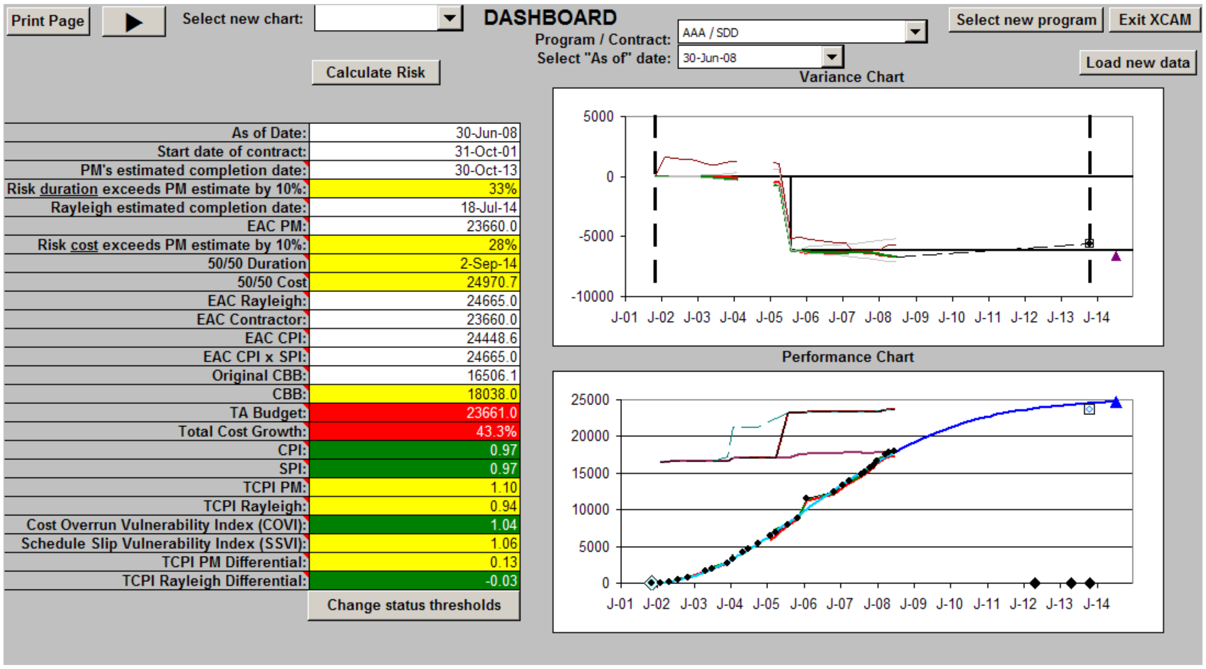
Since the Rayleigh model is not a cost estimation model, this Rayleigh analysis of the plan says nothing about the quality of the cost estimate underlying the final cost and schedule in the contract or the prospective offer. It simply assesses, given that cost and schedule and the associated plan, whether that plan is realistic (less risky) or unrealistic (risky).

Because this analysis only assesses the rate at which the work is planned to be done, it can be used to assess contractor offers during source selection. It can also be used before actual cost data is gathered to assess the plan for accomplishing work under contract (e.g. for an integrated baseline review (IBR)). This makes the Rayleigh model a potentially powerful and versatile tool.

Finding 5: We updated our diagnostic software application to incorporate desired features

We updated the Executive's Cost and Schedule Analysis (XCASA) tool to incorporate desired new features. Figure 10, below, shows an example of the dashboard for the Executive Contract Analysis Module (XCAM) portion of the software application. Figures 11, 12, and 13, all below, show the performance chart feature, the variance chart feature, and the risk analysis feature, respectively.

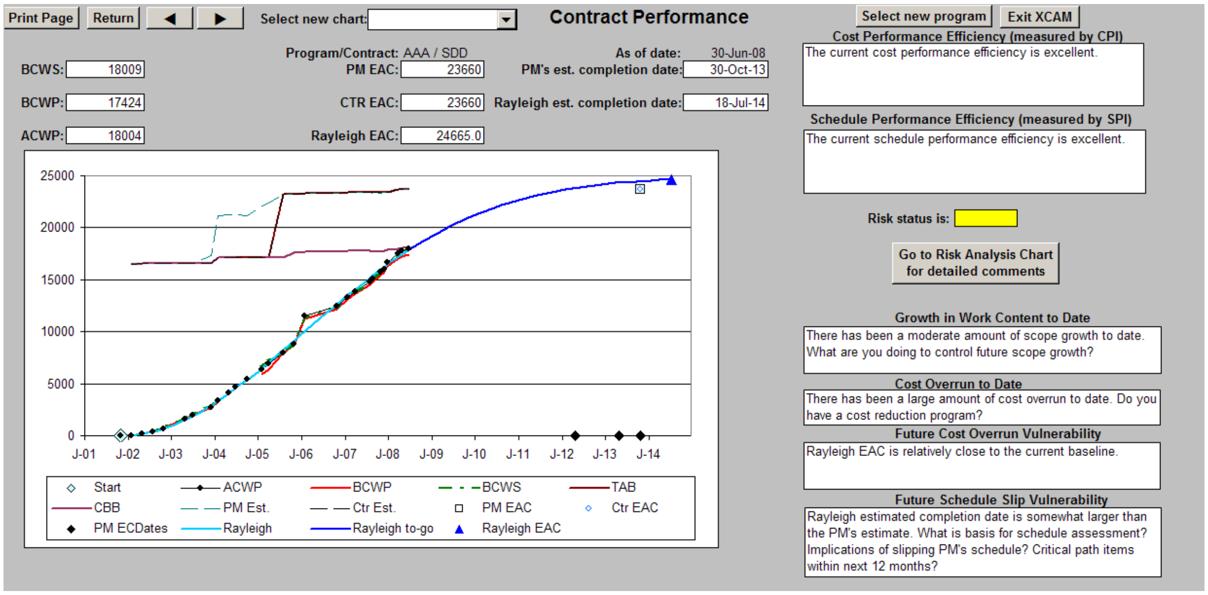
Figure 10. Sample dashboard from XCAM



Program / Contract: AAA / SDD
Select "As of" date: 30-Jun-08

As of Date:	30-Jun-08
Start date of contract:	31-Oct-01
PM's estimated completion date:	30-Oct-13
Risk duration exceeds PM estimate by 10%:	33%
Rayleigh estimated completion date:	18-Jul-14
EAC PM:	23660.0
Risk cost exceeds PM estimate by 10%:	28%
50/50 Duration	2-Sep-14
50/50 Cost	24970.7
EAC Rayleigh:	24665.0
EAC Contractor:	23660.0
EAC CPI:	24448.6
EAC CPI x SPI:	24665.0
Original CBB:	16506.1
CBB:	18038.0
TA Budget:	23661.0
Total Cost Growth:	43.3%
CPI:	0.97
SPI:	0.97
TCPI PM:	1.10
TCPI Rayleigh:	0.94
Cost Overrun Vulnerability Index (COVI):	1.04
Schedule Slip Vulnerability Index (SSVI):	1.06
TCPI PM Differential:	0.13
TCPI Rayleigh Differential:	-0.03

Figure 11. Sample performance chart from XCAM



Program/Contract: AAA / SDD
As of date: 30-Jun-08

BCWS: 18009
BCWP: 17424
ACWP: 18004

PM EAC: 23660
CTR EAC: 23660
Rayleigh EAC: 24665.0

PM's est. completion date: 30-Oct-13
Rayleigh est. completion date: 18-Jul-14

Cost Performance Efficiency (measured by CPI)

The current cost performance efficiency is excellent.

Schedule Performance Efficiency (measured by SPI)

The current schedule performance efficiency is excellent.

Risk status is: 🟨

Growth in Work Content to Date

There has been a moderate amount of scope growth to date. What are you doing to control future scope growth?

Cost Overrun to Date

There has been a large amount of cost overrun to date. Do you have a cost reduction program?

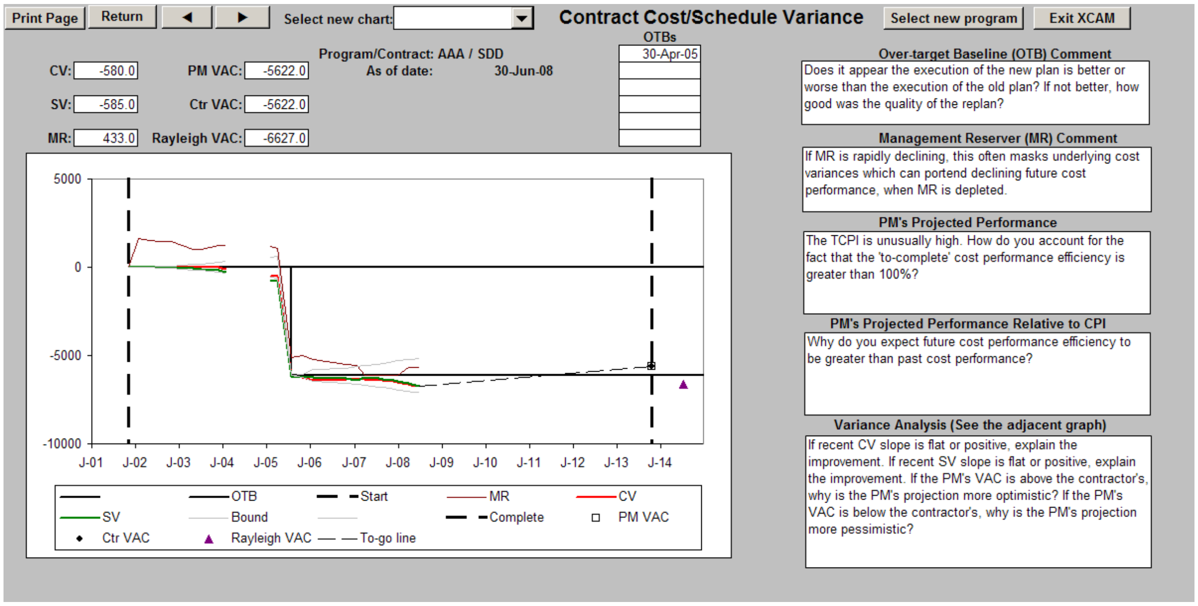
Future Cost Overrun Vulnerability

Rayleigh EAC is relatively close to the current baseline.

Future Schedule Slip Vulnerability

Rayleigh estimated completion date is somewhat larger than the PM's estimate. What is basis for schedule assessment? Implications of slipping PM's schedule? Critical path items within next 12 months?

Figure 12. Sample variance chart from XCAM



Program/Contract: AAA / SDD
As of date: 30-Jun-08

CV: -580.0
SV: -585.0
MR: 433.0
PM VAC: -5622.0
Ctr VAC: -5622.0
Rayleigh VAC: -6627.0

OTBs: 30-Apr-05

Over-target Baseline (OTB) Comment

Does it appear the execution of the new plan is better or worse than the execution of the old plan? If not better, how good was the quality of the replan?

Management Reserver (MR) Comment

If MR is rapidly declining, this often masks underlying cost variances which can portend declining future cost performance, when MR is depleted.

PM's Projected Performance

The TCPI is unusually high. How do you account for the fact that the 'to-complete' cost performance efficiency is greater than 100%?

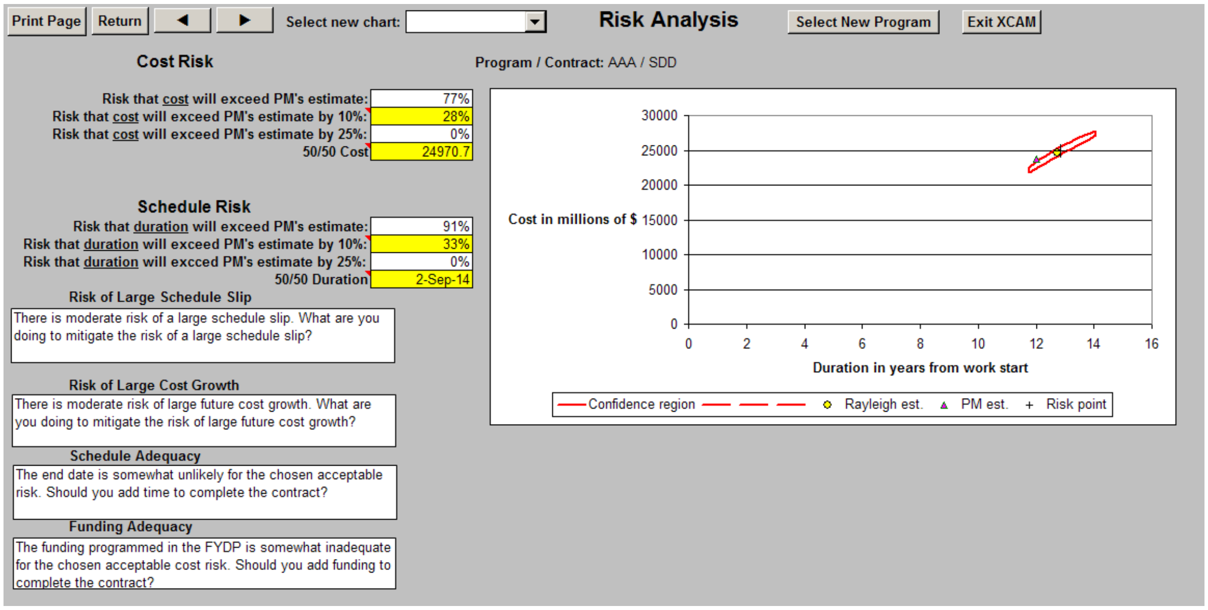
PM's Projected Performance Relative to CPI

Why do you expect future cost performance efficiency to be greater than past cost performance?

Variance Analysis (See the adjacent graph)

If recent CV slope is flat or positive, explain the improvement. If recent SV slope is flat or positive, explain the improvement. If the PM's VAC is above the contractor's, why is the PM's projection more optimistic? If the PM's VAC is below the contractor's, why is the PM's projection more pessimistic?

Figure 13. Sample risk analysis from XCAM



Program / Contract: AAA / SDD

Cost Risk

Risk that <u>cost</u> will exceed PM's estimate:	77%
Risk that <u>cost</u> will exceed PM's estimate by 10%:	28%
Risk that <u>cost</u> will exceed PM's estimate by 25%:	0%
50/50 Cost	24970.7

Schedule Risk

Risk that <u>duration</u> will exceed PM's estimate:	91%
Risk that <u>duration</u> will exceed PM's estimate by 10%:	33%
Risk that <u>duration</u> will excced PM's estimate by 25%:	0%
50/50 Duration	2-Sep-14

Risk of Large Schedule Slip

There is moderate risk of a large schedule slip. What are you doing to mitigate the risk of a large schedule slip?

Risk of Large Cost Growth

There is moderate risk of large future cost growth. What are you doing to mitigate the risk of large future cost growth?

Schedule Adequacy

The end date is somewhat unlikely for the chosen acceptable risk. Should you add time to complete the contract?

Funding Adequacy

The funding programmed in the FYDP is somewhat inadequate for the chosen acceptable cost risk. Should you add funding to complete the contract?

Figure 10 shows the dashboard, which contains summary data color-coded for threshold breaches. It includes summary risk information and miniature graphs of performance and variance. From this view, the user can "drill down" to the performance chart (figure 11), which contains expanded business insights, including: schedule performance efficiency, cost performance efficiency, an overall risk rating, an assessment of growth in work content, an assessment of cost overrun, an assessment of future cost overrun vulnerability, and an assessment of future schedule slip vulnerability. The insights will also tell the user if any critical EVMS data are missing.

The user can then "drill down" to the variance chart (figure 12), which can also shown OTBs to date. It includes business insights, including: any OTBs to date, an assessment of trends in the use of management reserve (MR), an assessment of the PM's projected performance, an assessment of the PM's projected performance relative to the CPI, and an assessment of the PM's variance at completion (VAC).

In addition, the user can "drill down" to the risk analysis chart (figure 13). This chart will have business insights, including: an assessment of the risk of a large schedule slip, an assessment of the risk of large future cost growth, an assessment of schedule adequacy for the desired level of acceptable risk, and an assessment of funding adequacy for the desired level of acceptable risk.

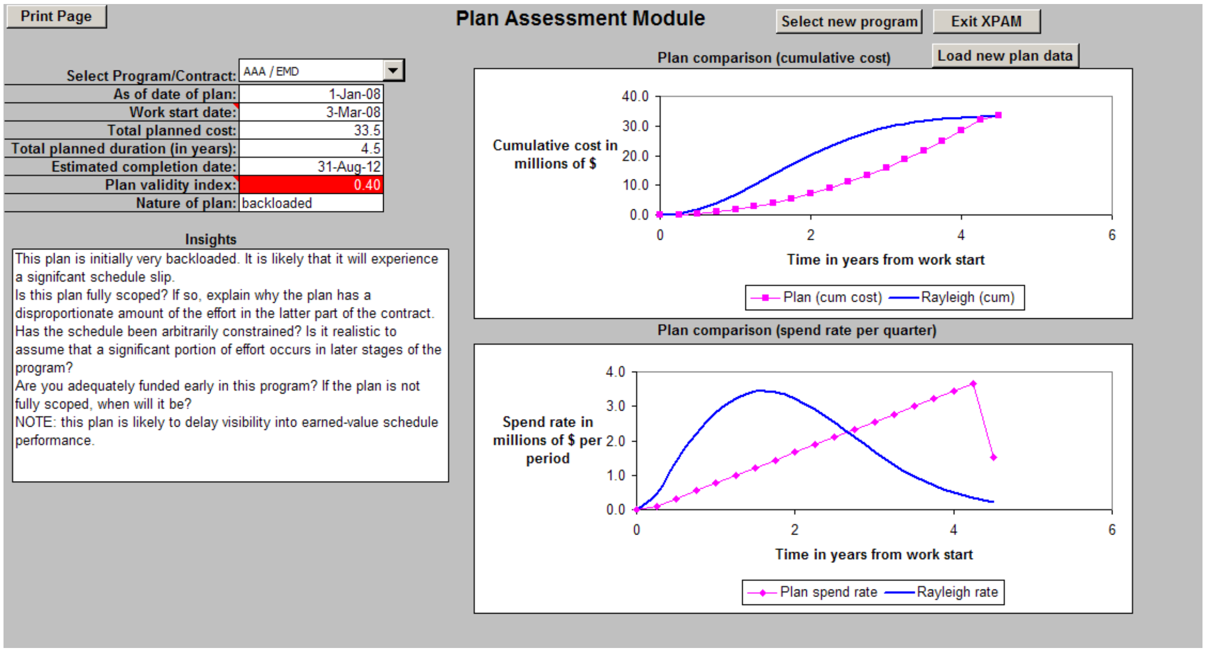
This module works with minimal information. It needs:

- reports of actual costs with their associated "as of" dates,
- a date when work started under the contract,
- a total allocated budget, and
- an estimated completion date.

Additional EVM data enhance the quality of the forecasts, but are not absolutely necessary. Further, comparison of Rayleigh forecast trends and EVM-generated forecast trends may provide an indication of the quality of the contractor's EVMS and its associated reports.

We have also updated the Executive Plan Assessment Module (XPAM). Figure 14 below shows an example of this module.

Figure 14. Sample of plan assessment module



Select Program/Contract: AAA / EMD
As of date of plan: 1-Jan-08
Work start date: 3-Mar-08
Total planned cost: 33.5
Total planned duration (in years): 4.5
Estimated completion date: 31-Aug-12
Plan validity index: 0.40
Nature of plan: backloaded

Insights

This plan is initially very backloaded. It is likely that it will experience a signifcant schedule slip.
Is this plan fully scoped? If so, explain why the plan has a disproportionate amount of the effort in the latter part of the contract.
Has the schedule been arbitrarily constrained? Is it realistic to assume that a significant portion of effort occurs in later stages of the program?
Are you adequately funded early in this program? If the plan is not fully scoped, when will it be?
NOTE: this plan is likely to delay visibility into earned-value schedule performance.

This display contains a graph of the cumulative Rayleigh curve, and a graph of the Rayleigh spend rate curve, obtained by numerically differentiating the cumulative Rayleigh curve. By numerically integrating the area between the cumulative Rayleigh curve and the plan for accomplishing work, we show the PVI, which is a metric of the realism or the relative risk associated with the plan.

This module may be used to assess contractor offers during source selection, to assess the realism of program RDT&E funding profiles, or to assess contract plan realism or plan risk before any actual costs have been reported. Once three reports of actual cost have been received, the user may start using the XCAM module to assess actual contract execution.

This module can operate with minimal information. It needs:

- a date when work under contract is to commence,

- a date when work under contract is supposed to end,
- a total value for the work under contract, and
- the contractor's plan for accomplishing the work, expressed in budget dollars per period (month, quarter, or year).

To summarize, we have improved our Rayleigh algorithm and its associated code in XCASA. Specifically, our updated XCASA:

- Contains code that can graph OTBs
- Contains code to assess overall contract and schedule risk in a contract
- Contains code and features to enhance the user interface and utility for the user
- Contains expanded and more robust business insights
- Contains an updated plan assessment module with expanded business insights
- Includes detailed instructions for using XCASA (appendix C)
- Includes detailed instructions for loading data (appendix D)

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Recommendations and conclusion

Recommendations

We used the Rayleigh model as a basis for our software application. In particular, the XCAM model should be used to manage contract execution. This module produces multiple estimates of cost and schedule. In general, we recommend using the Rayleigh estimate of cost and schedule and the associated risk analysis, subject to the caveats noted in the finding describing "rules of thumb" when applying the Rayleigh model.

We particularly recommend using the business insights feature of XCAM to prompt questions and possible issues arising during contract execution. We think the use of XCAM will also help the executive prioritize his effort as he oversees and manages a large acquisition portfolio.

An additional benefit of the XCAM module is that it can be used to track cost and duration prediction profiles or trends over the course of contract execution. If the Rayleigh trend separates from the other estimated trends, this may indicate that the EVMS reporting is faulty. Also XCAM will generate estimates and risk analysis even when EVM data are missing. The module will also indicate when EVM information is missing or when there have been gaps in the reporting of EVM data.

We recommend using the XPAM application, designed on the basis of the Rayleigh model, to assess plans. This module can be used to assess contractor offers during source selection. The model can also be used after contract award and commencement of execution to assess the validity of the contractor's plan, even before any actual cost data have been reported. An additional use of the module would be to use it to assess the validity of program RDT&E funding profiles.

We also recommend that the Navy fund further study and an additional upgrade to the software application, which could be improved to upgrade the data import interface with Navy databases. Further, the application could be recoded in other programming languages to make it possible to access and use it on the web. Making the tool web-based would enhance user interface and accessibility. We would also recommend further study to build a simulations capability into the application to make it an even more powerful tool for the executive decision-maker.

Conclusion

We have developed a new algorithm to numerically minimize the sum of residuals, using the Rayleigh model, that we call NRP. This model for estimating cost and schedule generates better estimates of both cost and schedule, and the risks inherent to both cost and schedule during contract execution.

Subject to noted caveats, the Rayleigh model is a powerful tool for analyzing contract execution and plan validity. In addition to diagnosing cost and schedule issues in a contract, the Rayleigh model can also diagnose possibly faulty EVMS implementation. In short, the Rayleigh model is a very useful addition to the decision-makers toolkit that provides him with very early warning of potential plan or contract execution issues.

Appendix A: Mathematical relationships

The derivation of the relationship between α and time of peak expenditure rate, which we denote as t_p, is shown in the following steps:

$c(t) = d(1 - e^{-\alpha t^2})$ (the Rayleigh cumulative cost function)

$\dfrac{dc}{dt} = r(t) = 2\alpha d t e^{-\alpha t^2}$ (the expenditure rate function)

$2\alpha d e^{-\alpha t_p^2} - 4\alpha^2 d t_p^2 e^{-\alpha t_p^2} = 0$ (first order conditions for the rate function)

$2 - 4\alpha t_p^2 = 0$ (the simplification of the first order condition)

$t_p = \sqrt{\dfrac{.5}{\alpha}}$ (the solution of the above for t_p)

We require that α be strictly positive. In this case, a quick check of second order conditions confirms the rate function is concave. So, the result of the first-order-conditions for optimization does indeed result in a maximum. The time t_p is the time at which the peak expenditure rate occurs. This also corresponds to the inflexion point of the cumulative Rayleigh curve.

The demonstration of the relationship between α and the completion time, which we denote t_f,, is derived below:

$c_f = .99d$ (stipulated tolerance for relating *d* and final realized cost)

$c_f = .99d = d(1 - e^{-\alpha t_f^2})$ (final realized cost occurs at completion time)

$e^{-\alpha t_f^2} = .01$ (simplification of the above)

$-\alpha t_f{}^2 = \ln(.01)$ (simplification of the above)

$\alpha = \dfrac{-\ln(.01)}{t_f^2}$ (solution of relationship between α and t_f)

$t_f = \sqrt{-\ln(.01)/\alpha}$ (expressing contract duration in terms of α)

This clearly shows the reciprocal relation between the shape parameter and the completion time or the time of contract duration. The larger the shape parameter is, the smaller the completion time will be, and vice versa.

Appendix B: Contracting for earned value management (EVM)

EVM depends on disciplined management of the performance measurement baseline (PMB), the time-phased plan for consumption of resources needed to complete the contract. The PMB is derived from the contract value at the cost level, excluding profit or fee. DoD policy requires that EVM be used above a specified dollar threshold (currently $20M for development contracts and $50M for production) for cost type contracts and for fixed price contracts with incentive sharing arrangements.

The government bears the highest risk on any cost-type contract, which requires that the government pay all allowable and allocable costs incurred by the contractor.

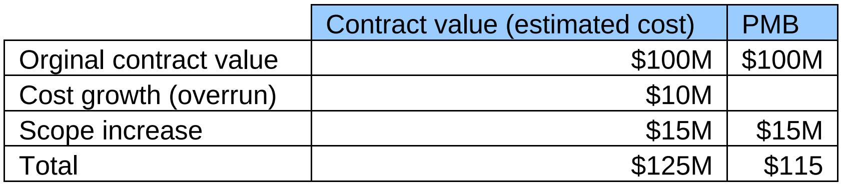
When EVM was implemented as DoD policy in the late 1960s, Service acquisition headquarters organizations routinely examined major contracts to ensure that management requirements were incorporated properly, and there was continuing oversight to monitor proper use. Since then, management has become decentralized and the procurement community has lost sight of the distinction between contract cost growth due to scope increases and contract overruns.

A 2004 study by CNA of Navy cost growth experience found that overall contract cost growth is nearly level, but with an increasing proportion of "scope" changes offsetting declining overruns. This suggests a lack of discipline in maintaining the distinction between scope changes and cost overruns, especially in non-incentive contracts.

The following example illustrates the issue. Assume a Cost Plus Fixed Fee (CPFF) contract is awarded at an estimated cost of

$100M. During contract performance, a cost overrun is identified through earned value analysis that indicates cost at completion will be $110M. At the same time, the contract is modified to add $15M in added scope. When the contract modification is issued, the new estimated cost will be $125M. However, for performance measurement purposes, the PMB should be $115M. See figure 1 below.

Figure 15. Contracting for EVM



	Contract value (estimated cost)	PMB
Orginal contract value	$100M	$100M
Cost growth (overrun)	$10M	
Scope increase	$15M	$15M
Total	$125M	$115

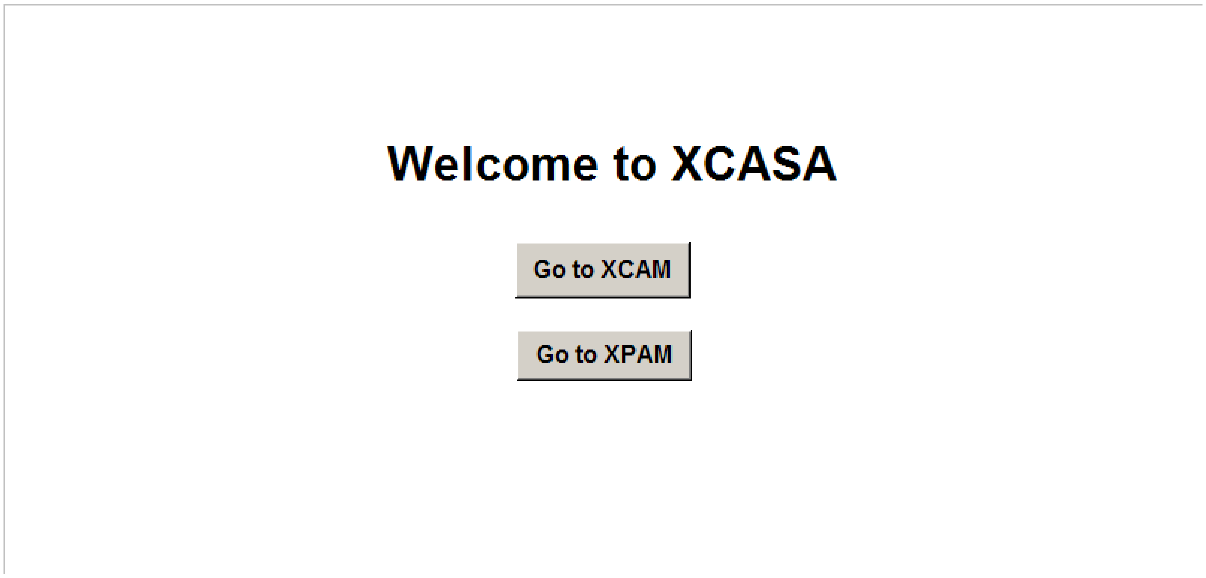
Because the government is obligated to pay all the allowable cost, procurement organizations that do not appreciate the need for disciplined baseline management may not understand why the contract modification should distinguish the new estimated cost from the PMB value. This situation is not as prevalent in incentive-type contracts, presumably because the target cost and shared cost risk instill a higher regard for maintaining baseline discipline. Just as in an incentive contract, if cost overruns degrade the PMB as a basis for management and measurement, the contractor and procuring activity should favorably consider using an over-target baseline to restore meaningful performance measurement.

Appendix C: User's guide for XCASA

Guide for XCAM

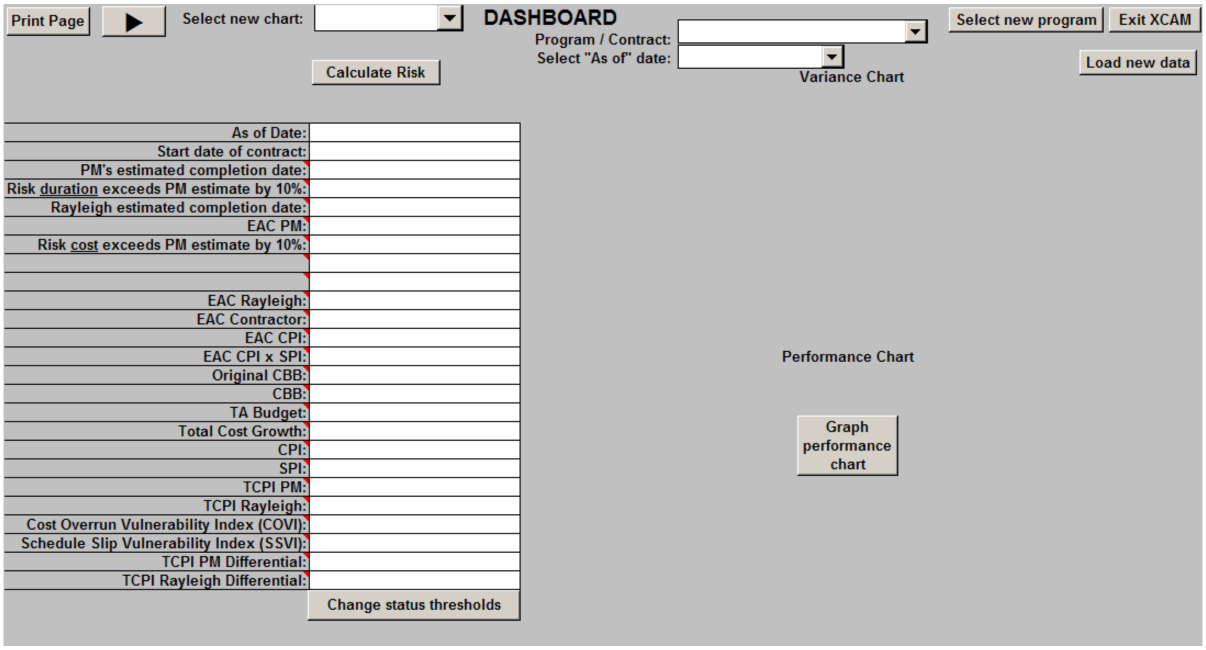
Open the file for XCASAv1. xls. Enable macros. You should see a view like the one shown below in figure 2.

Figure 16. XCASA opening page



Press the button labeled "Go to XCAM." This will take you to the dashboard shown below in figure 3.

Figure 17. XCASA dashboard at entry point



The summary data are labeled. You can run your "mouse" over labels identified with a red triangle to read a short description of that data element.

To select a program, select the program/contract drop-down and choose the desired program. A message box will appear informing you that it may take a few minutes to complete a series of calculations. Press "OK" for the calculations to begin.

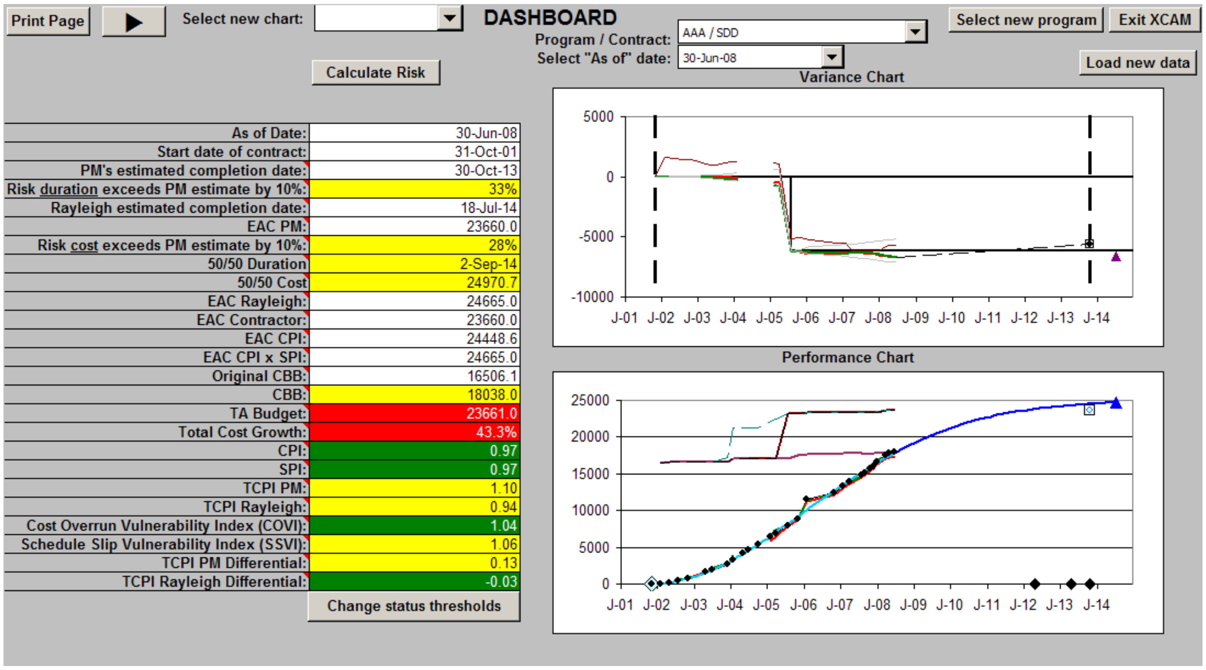
When calculations are complete, another message box will appear informing you that the calculations are finished. Press "OK."

Data elements will be filled in with appropriate threshold color codes. Also, a miniature variance chart will appear. Press the "Graph performance chart" button and a miniature performance chart will appear.

Press the "Calculate risk" button and risk data elements will be filled with appropriate threshold color codes.

See figure 4, below, for a view of the dashboard when these steps are complete.

Figure 18. View of the dashboard when steps are completed

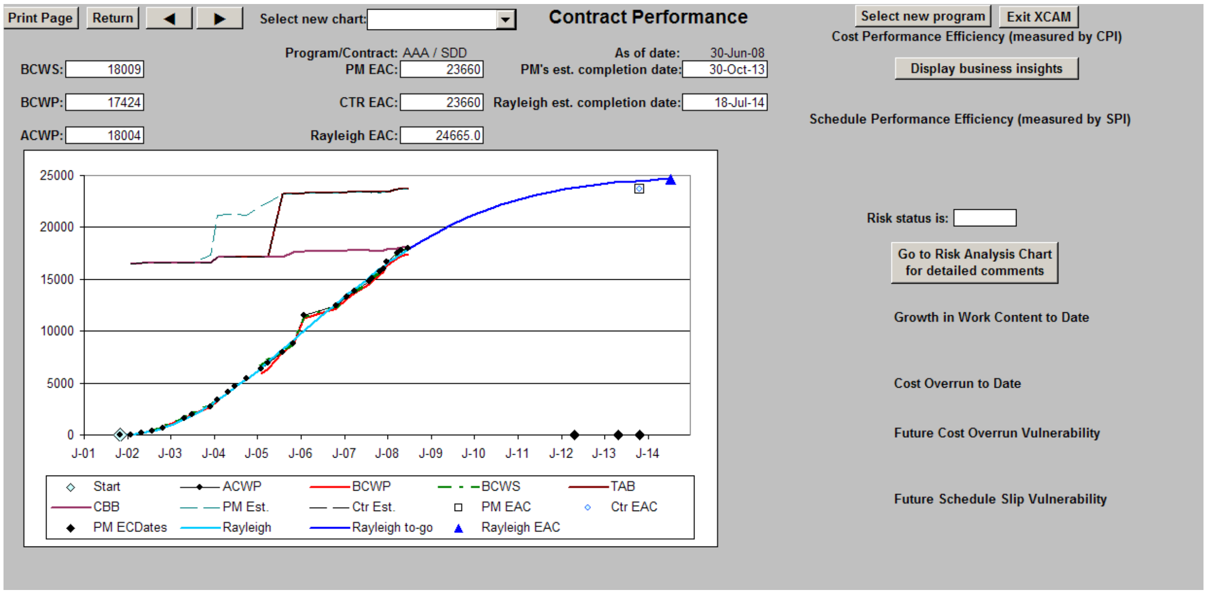


Program / Contract: AAA / SDD
Select "As of" date: 30-Jun-08

As of Date:	30-Jun-08
Start date of contract:	31-Oct-01
PM's estimated completion date:	30-Oct-13
Risk duration exceeds PM estimate by 10%:	33%
Rayleigh estimated completion date:	18-Jul-14
EAC PM:	23660.0
Risk cost exceeds PM estimate by 10%:	28%
50/50 Duration	2-Sep-14
50/50 Cost	24970.7
EAC Rayleigh:	24665.0
EAC Contractor:	23660.0
EAC CPI:	24448.6
EAC CPI x SPI:	24665.0
Original CBB:	16506.1
CBB:	18038.0
TA Budget:	23661.0
Total Cost Growth:	43.3%
CPI:	0.97
SPI:	0.97
TCPI PM:	1.10
TCPI Rayleigh:	0.94
Cost Overrun Vulnerability Index (COVI):	1.04
Schedule Slip Vulnerability Index (SSVI):	1.06
TCPI PM Differential:	0.13
TCPI Rayleigh Differential:	-0.03

Now press either right arrow or press the drop-down menu for "Select new chart" and choose "Performance Chart." The view shown below in figure 5 is a screen shot of this "drill-down" upon entry.

Figure 19. View of the performance chart upon entry



Contract Performance

Program/Contract: AAA / SDD
As of date: 30-Jun-08
BCWS: 18009
PM EAC: 23660
PM's est. completion date: 30-Oct-13
BCWP: 17424
CTR EAC: 23660
Rayleigh est. completion date: 18-Jul-14
ACWP: 18004
Rayleigh EAC: 24665.0

Cost Performance Efficiency (measured by CPI)

Schedule Performance Efficiency (measured by SPI)

Risk status is:

Growth in Work Content to Date

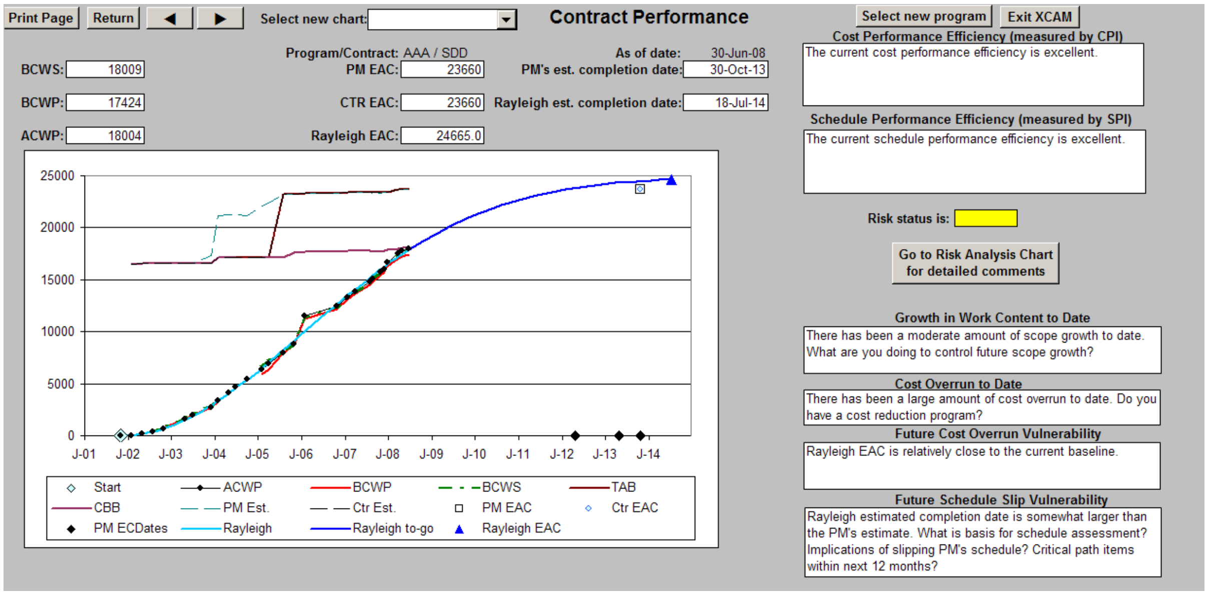
Cost Overrun to Date

Future Cost Overrun Vulnerability

Future Schedule Slip Vulnerability

This view has a larger scale version of the performance chart that is color-coded and has a legend. Press the "Display business insights" button. Figure 6, below, shows the view when this is complete.

Figure 20. View of the completed performance chart



Contract Performance

Program/Contract: AAA / SDD
As of date: 30-Jun-08
BCWS: 18009
PM EAC: 23660
PM's est. completion date: 30-Oct-13
BCWP: 17424
CTR EAC: 23660
Rayleigh est. completion date: 18-Jul-14
ACWP: 18004
Rayleigh EAC: 24665.0

Cost Performance Efficiency (measured by CPI)
The current cost performance efficiency is excellent.

Schedule Performance Efficiency (measured by SPI)
The current schedule performance efficiency is excellent.

Risk status is: 🟨

Growth in Work Content to Date
There has been a moderate amount of scope growth to date. What are you doing to control future scope growth?

Cost Overrun to Date
There has been a large amount of cost overrun to date. Do you have a cost reduction program?

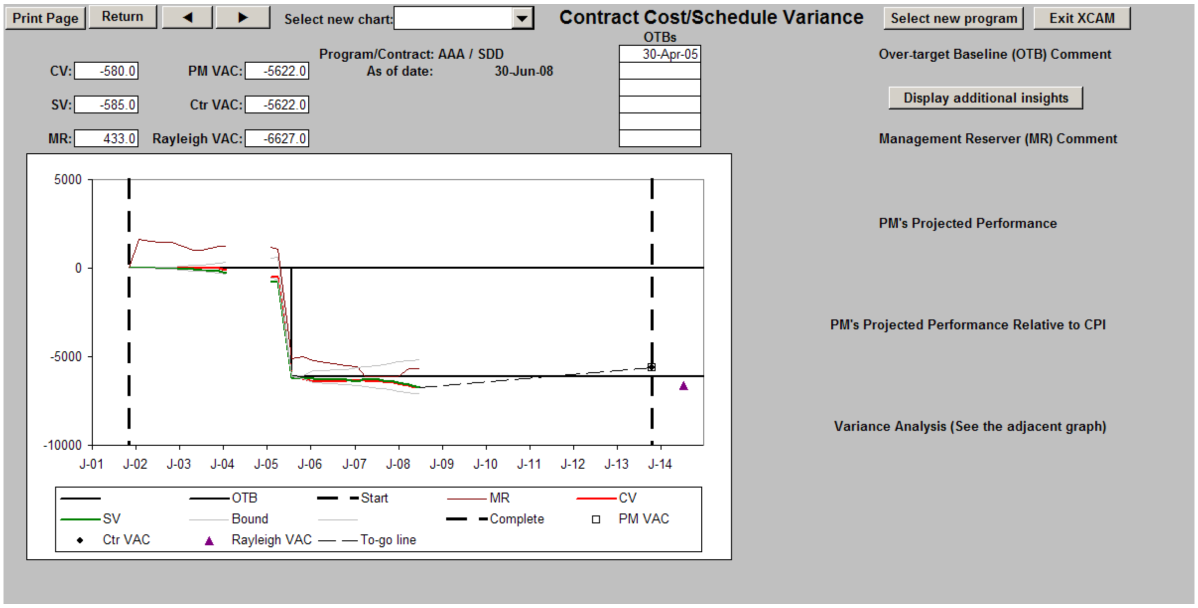
Future Cost Overrun Vulnerability
Rayleigh EAC is relatively close to the current baseline.

Future Schedule Slip Vulnerability
Rayleigh estimated completion date is somewhat larger than the PM's estimate. What is basis for schedule assessment? Implications of slipping PM's schedule? Critical path items within next 12 months?

The view now has a set of business insights keyed to contract performance to date. It also has a color code for the overall risk status of the contract. There is an optional button for going directly to the "Risk analysis" view to see risk insights and analysis.

Now press the right arrow or press the "Select new chart" dropdown and choose "Variance Chart." Figure 7, below, show the initial view of the variance chart.

Figure 21. Initial view of the variance chart



Contract Cost/Schedule Variance

CV: -580.0

SV: -585.0

MR: 433.0

PM VAC: -5622.0

Ctr VAC: -5622.0

Rayleigh VAC: -6627.0

Program/Contract: AAA / SDD

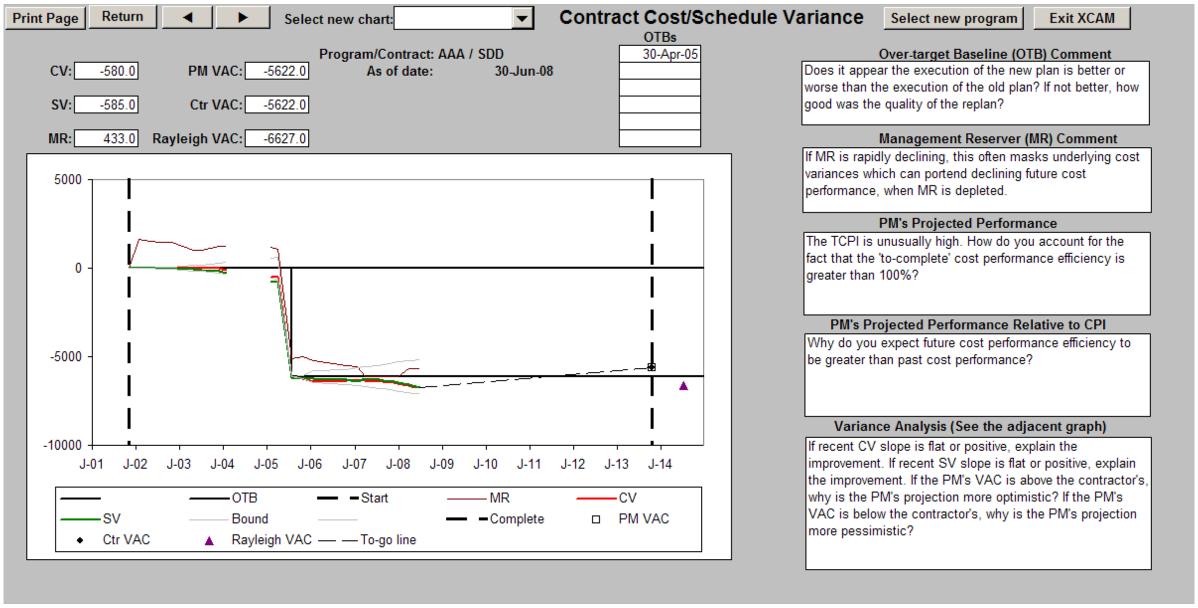
As of date: 30-Jun-08

OTBs: 30-Apr-05

This view shows a larger-scale version of the variance chart that is color coded and has a legend.

Press the "Display additional insights" button. Figure 8, below , shows the completed view of the variance chart.

Figure 22. Completed view of the variance chart



Contract Cost/Schedule Variance

Program/Contract: AAA / SDD
As of date: 30-Jun-08

CV: -580.0
SV: -585.0
MR: 433.0
PM VAC: -5622.0
Ctr VAC: -5622.0
Rayleigh VAC: -6627.0

OTBs: 30-Apr-05

Over-target Baseline (OTB) Comment

Does it appear the execution of the new plan is better or worse than the execution of the old plan? If not better, how good was the quality of the replan?

Management Reserver (MR) Comment

If MR is rapidly declining, this often masks underlying cost variances which can portend declining future cost performance, when MR is depleted.

PM's Projected Performance

The TCPI is unusually high. How do you account for the fact that the 'to-complete' cost performance efficiency is greater than 100%?

PM's Projected Performance Relative to CPI

Why do you expect future cost performance efficiency to be greater than past cost performance?

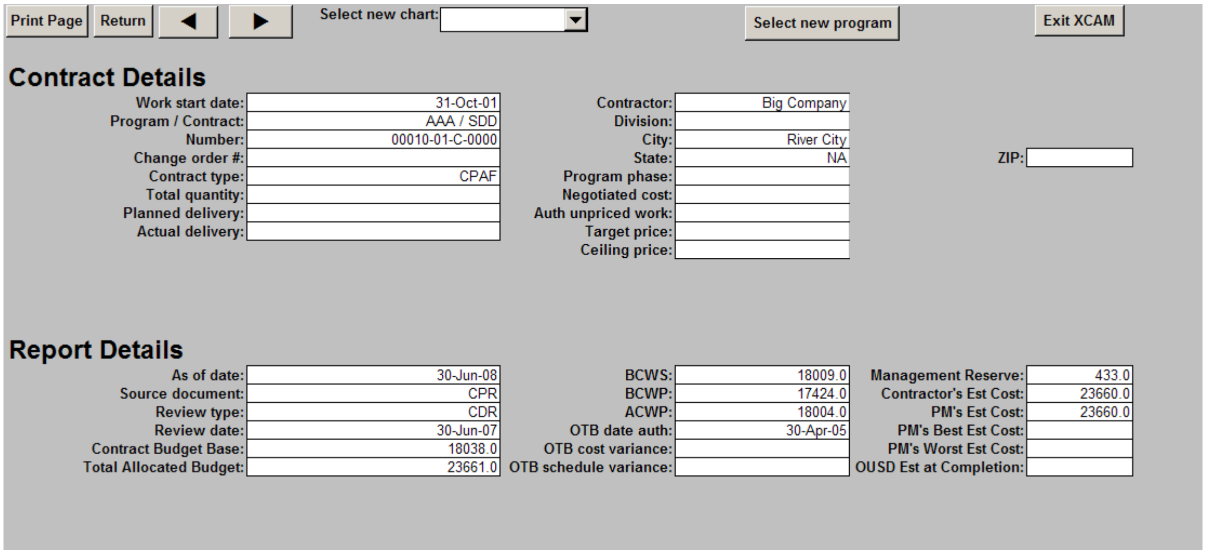
Variance Analysis (See the adjacent graph)

If recent CV slope is flat or positive, explain the improvement. If recent SV slope is flat or positive, explain the improvement. If the PM's VAC is above the contractor's, why is the PM's projection more optimistic? If the PM's VAC is below the contractor's, why is the PM's projection more pessimistic?

This view shows variance data. If an OTB has occurred, the data will be noted and the graph will show the standard "step" function to indicate an OTB. The additional business insights are related to analysis of the OTB (if it has occurred), insights related to projected performance and comments related to variance and VAC calculations.

Press the right arrow or press the "Select new chart" drop-down and choose "Contract details chart." Figure 9, below, shows the view of the contract details chart.

Figure 23. View of the contract details chart



Contract Details

Work start date: 31-Oct-01
Program / Contract: AAA / SDD
Number: 00010-01-C-0000
Change order #:
Contract type: CPAF
Total quantity:
Planned delivery:
Actual delivery:

Contractor: Big Company
Division:
City: River City
State: NA
ZIP:
Program phase:
Negotiated cost:
Auth unpriced work:
Target price:
Ceiling price:

Report Details

As of date: 30-Jun-08
Source document: CPR
Review type: CDR
Review date: 30-Jun-07
Contract Budget Base: 18038.0
Total Allocated Budget: 23661.0

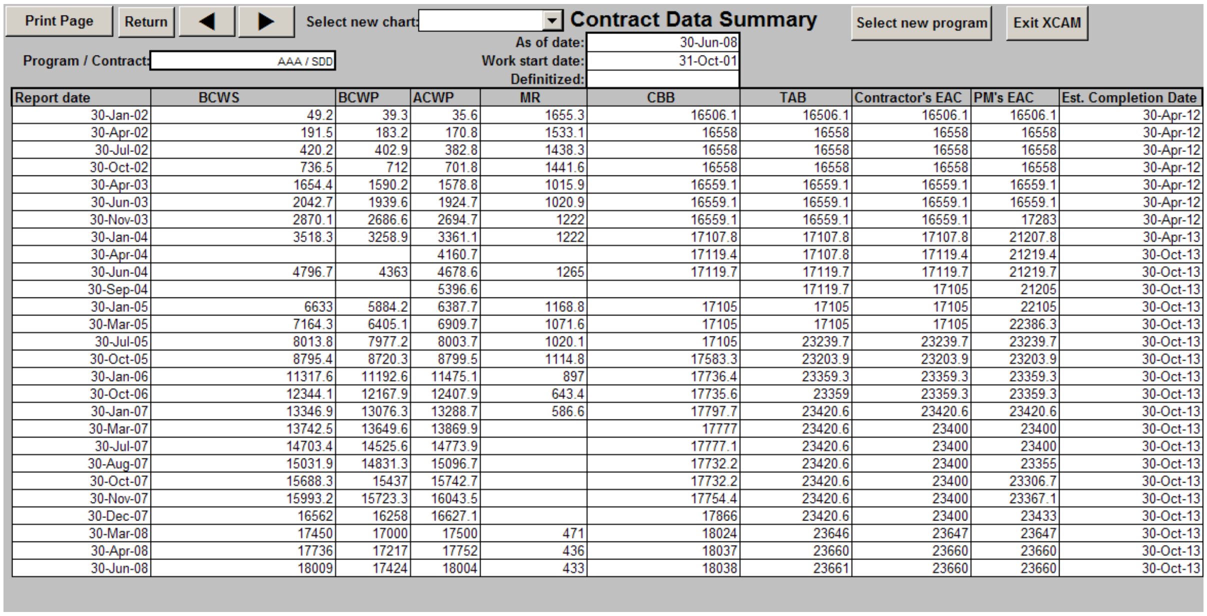
BCWS: 18009.0
BCWP: 17424.0
ACWP: 18004.0
OTB date auth: 30-Apr-05
OTB cost variance:
OTB schedule variance:

Management Reserve: 433.0
Contractor's Est Cost: 23660.0
PM's Est Cost: 23660.0
PM's Best Est Cost:
PM's Worst Est Cost:
OUSD Est at Completion:

This view shows in tabular form details about the contract and the report for the user's inspection.

Press the right arrow or press the "Select new chart" drop-down and choose "Contract data chart." Figure 10, below, shows the view of the contract data chart.

Figure 24. View of the contract data chart



Contract Data Summary

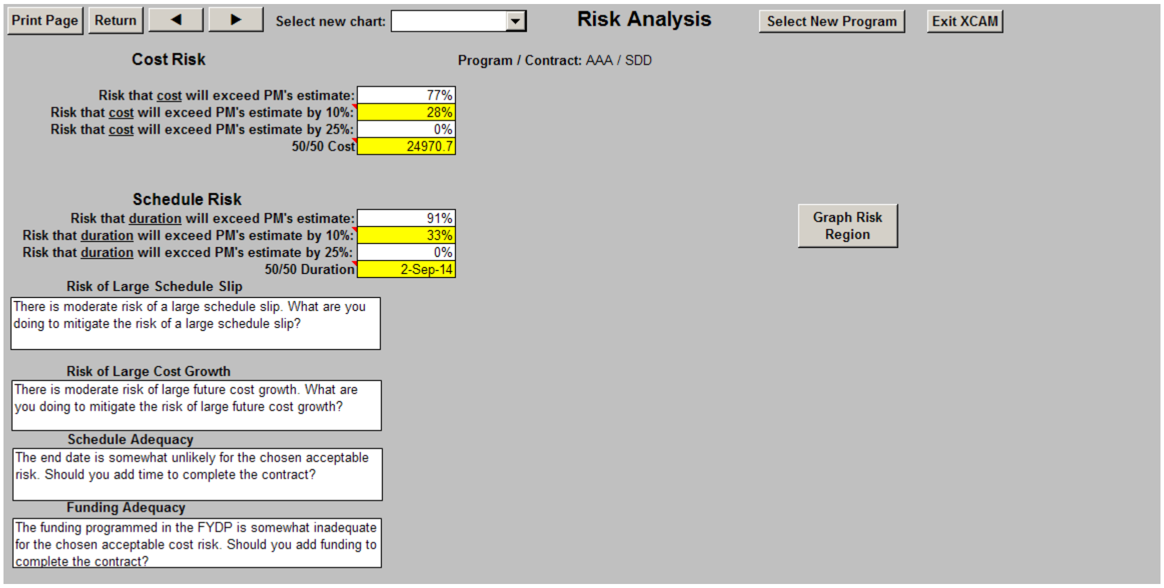
Program / Contract: AAA / SDD
As of date: 30-Jun-08
Work start date: 31-Oct-01
Definitized:

Report date	BCWS	BCWP	ACWP	MR	CBB	TAB	Contractor's EAC	PM's EAC	Est. Completion Date
30-Jan-02	49.2	39.3	35.6	1655.3	16506.1	16506.1	16506.1	16506.1	30-Apr-12
30-Apr-02	191.5	183.2	170.8	1533.1	16558	16558	16558	16558	30-Apr-12
30-Jul-02	420.2	402.9	382.8	1438.3	16558	16558	16558	16558	30-Apr-12
30-Oct-02	736.5	712	701.8	1441.6	16558	16558	16558	16558	30-Apr-12
30-Apr-03	1654.4	1590.2	1578.8	1015.9	16559.1	16559.1	16559.1	16559.1	30-Apr-12
30-Jun-03	2042.7	1939.6	1924.7	1020.9	16559.1	16559.1	16559.1	16559.1	30-Apr-12
30-Nov-03	2870.1	2686.6	2694.7	1222	16559.1	16559.1	16559.1	17283	30-Apr-12
30-Jan-04	3518.3	3258.9	3361.1	1222	17107.8	17107.8	17107.8	21207.8	30-Apr-13
30-Apr-04			4160.7		17119.4	17107.8	17119.4	21219.4	30-Oct-13
30-Jun-04	4796.7	4363	4678.6	1265	17119.7	17119.7	17119.7	21219.7	30-Oct-13
30-Sep-04			5396.6			17119.7	17105	21205	30-Oct-13
30-Jan-05	6633	5884.2	6387.7	1168.8	17105	17105	17105	22105	30-Oct-13
30-Mar-05	7164.3	6405.1	6909.7	1071.6	17105	17105	17105	22386.3	30-Oct-13
30-Jul-05	8013.8	7977.2	8003.7	1020.1	17105	23239.7	23239.7	23239.7	30-Oct-13
30-Oct-05	8795.4	8720.3	8799.5	1114.8	17583.3	23203.9	23203.9	23203.9	30-Oct-13
30-Jan-06	11317.6	11192.6	11475.1	897	17736.4	23359.3	23359.3	23359.3	30-Oct-13
30-Oct-06	12344.1	12167.9	12407.9	643.4	17735.6	23359	23359.3	23359.3	30-Oct-13
30-Jan-07	13346.9	13076.3	13288.7	586.6	17797.7	23420.6	23420.6	23420.6	30-Oct-13
30-Mar-07	13742.5	13649.6	13869.9		17777	23420.6	23400	23400	30-Oct-13
30-Jul-07	14703.4	14525.6	14773.9		17777.1	23420.6	23400	23400	30-Oct-13
30-Aug-07	15031.9	14831.3	15096.7		17732.2	23420.6	23400	23355	30-Oct-13
30-Oct-07	15688.3	15437	15742.7		17732.2	23420.6	23400	23306.7	30-Oct-13
30-Nov-07	15993.2	15723.3	16043.5		17754.4	23420.6	23400	23367.1	30-Oct-13
30-Dec-07	16562	16258	16627.1		17866	23420.6	23400	23433	30-Oct-13
30-Mar-08	17450	17000	17500	471	18024	23646	23647	23647	30-Oct-13
30-Apr-08	17736	17217	17752	436	18037	23660	23660	23660	30-Oct-13
30-Jun-08	18009	17424	18004	433	18038	23661	23660	23660	30-Oct-13

This view has a tabular view of key data to date. The user can quickly see if there are missing data. The data are sorted from earliest "as of" date to the most recent date.

Press the right arrow or press the "Select new chart" drop-down and select "Risk analysis chart." Figure 11, below, shows the initial view of this chart.

Figure 25. Initial view of risk analysis chart



Risk Analysis

Program / Contract: AAA / SDD

Cost Risk

Risk that cost will exceed PM's estimate:	77%
Risk that cost will exceed PM's estimate by 10%:	28%
Risk that cost will exceed PM's estimate by 25%:	0%
50/50 Cost	24970.7

Schedule Risk

Risk that duration will exceed PM's estimate:	91%
Risk that duration will exceed PM's estimate by 10%:	33%
Risk that duration will excced PM's estimate by 25%:	0%
50/50 Duration	2-Sep-14

Risk of Large Schedule Slip

There is moderate risk of a large schedule slip. What are you doing to mitigate the risk of a large schedule slip?

Risk of Large Cost Growth

There is moderate risk of large future cost growth. What are you doing to mitigate the risk of large future cost growth?

Schedule Adequacy

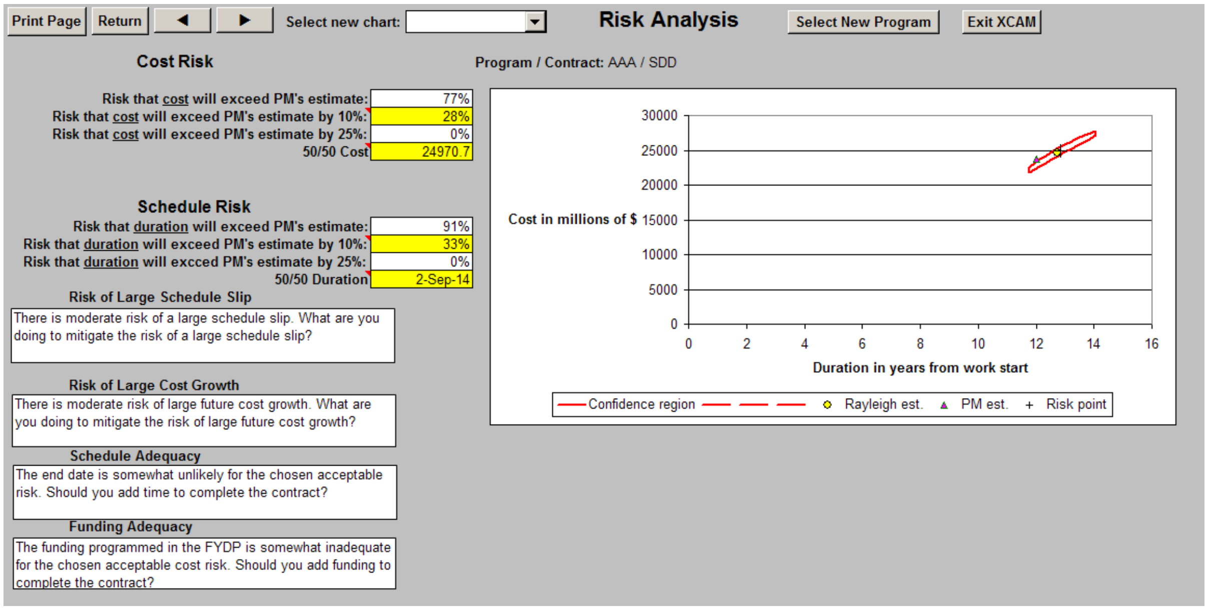
The end date is somewhat unlikely for the chosen acceptable risk. Should you add time to complete the contract?

Funding Adequacy

The funding programmed in the FYDP is somewhat inadequate for the chosen acceptable cost risk. Should you add funding to complete the contract?

This chart replicates risk calculation with color-coded thresholds. It also contains business insights related to cost and schedule risk. There is an option button to see the graph of the risk region. If you press this button, figure 12, below, shows the completed view of the risk analysis chart.

Figure 26. Completed view of the risk analysis chart



Cost Risk

Program / Contract: AAA / SDD

Risk that cost will exceed PM's estimate:	77%
Risk that cost will exceed PM's estimate by 10%:	28%
Risk that cost will exceed PM's estimate by 25%:	0%
50/50 Cost	24970.7

Schedule Risk

Risk that duration will exceed PM's estimate:	91%
Risk that duration will exceed PM's estimate by 10%:	33%
Risk that duration will excced PM's estimate by 25%:	0%
50/50 Duration	2-Sep-14

Risk of Large Schedule Slip

There is moderate risk of a large schedule slip. What are you doing to mitigate the risk of a large schedule slip?

Risk of Large Cost Growth

There is moderate risk of large future cost growth. What are you doing to mitigate the risk of large future cost growth?

Schedule Adequacy

The end date is somewhat unlikely for the chosen acceptable risk. Should you add time to complete the contract?

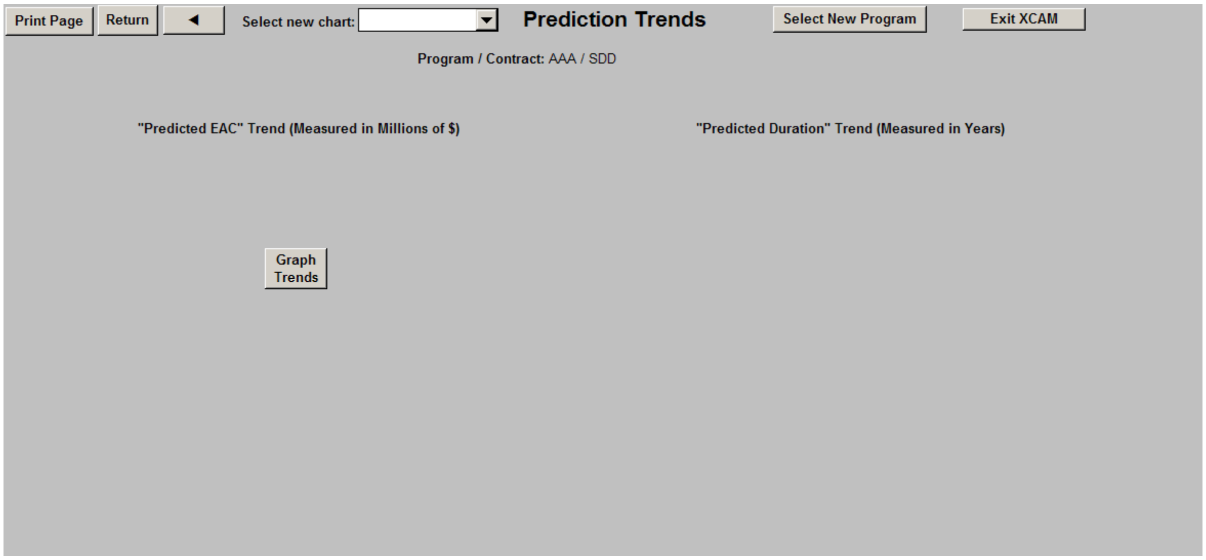
Funding Adequacy

The funding programmed in the FYDP is somewhat inadequate for the chosen acceptable cost risk. Should you add funding to complete the contract?

To the right, a graph of the risk region appears along with the Rayleigh estimate, the PM's estimate, and the risk point. The color code for the legend is below the graph.

Press the right arrow or press the "Select new chart" drop-down and choose the "Trends chart." Figure 13, below, shows the initial view of this chart.

Figure 27. Initial view of the prediction trends chart

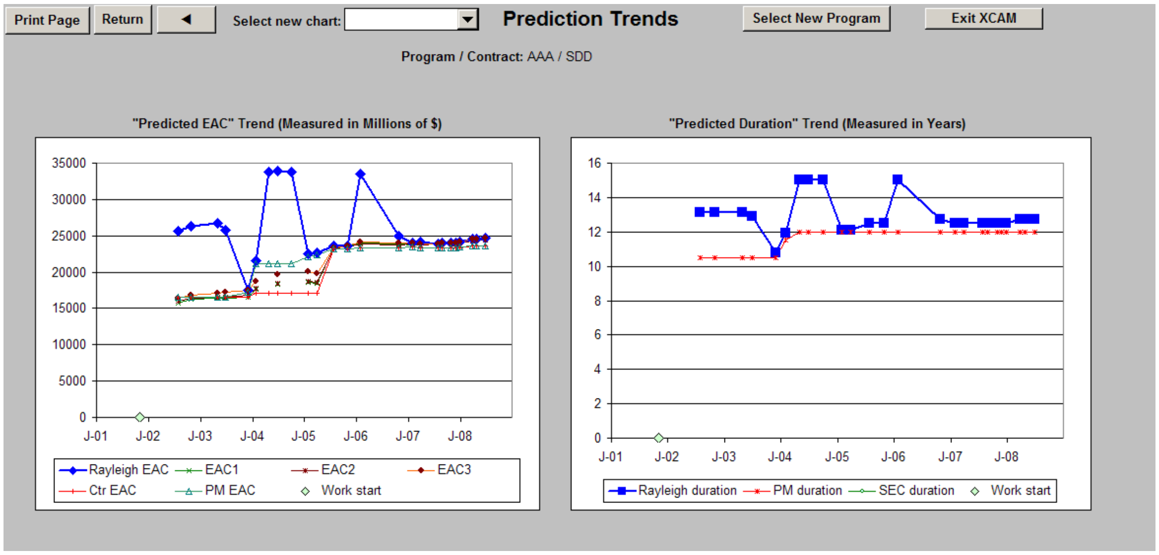


Prediction Trends

Program / Contract: AAA / SDD

If you wish, press the "Graph Trends" button. Figure 14, below, shows the completed view of prediction trends chart.

Figure 28. Completed view of the prediction trends chart



Prediction Trends

Program / Contract: AAA / SDD

The chart on the left shows the cost prediction profiles or trends for all estimation techniques. The chart on the right shows the duration prediction profiles in years for all estimation techniques. A large separation between the Rayleigh profiles of estimates and other profiles may be an indicator that the EVMS reporting system is faulty.

If you wish to see what the views for the same contract look like for earlier dates, press the "Select new chart" drop-down and choose the "Dashboard Chart." Then select "Select 'as of' date" drop-down and choose your desired date for the new view. Then, go through the same steps described above to see the new views.

If you wish to see a new program/contract, return to the Dashboard view as already described and press the "Select new program" button to clear the existing data. Then press the "program/contract" drop-down, select a new program/contract, and proceed as described above.

If you wish to print a view, press the "Print view" button on that view.

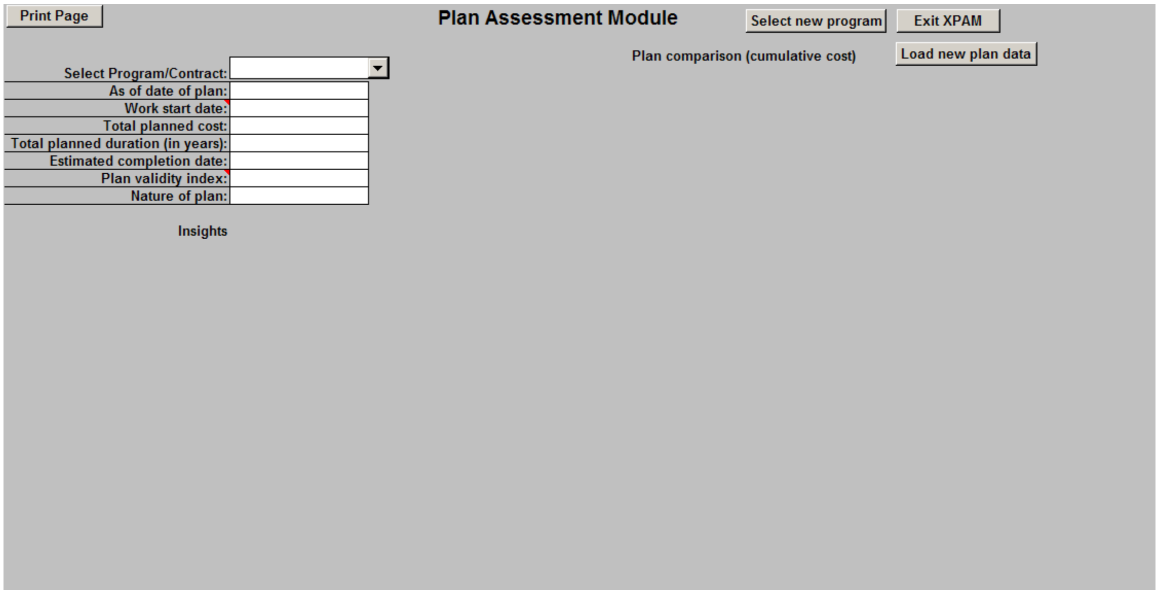
When you are done with XCAM, press the "Exit XCAM" button to clear existing data. This will also return you to the opening page. If you are done with XCASA, simple close the file and select "do not save."

Guide for XPAM

If you are in XCAM, press the exit XCAM button. If you are just starting, open the application as described above and enable the macros.

On the opening page, select the "Go to XPAM" button. Figure 15, below, shows you the initial view of the plan assessment module.

Figure 29. Initial view of plan assessment module

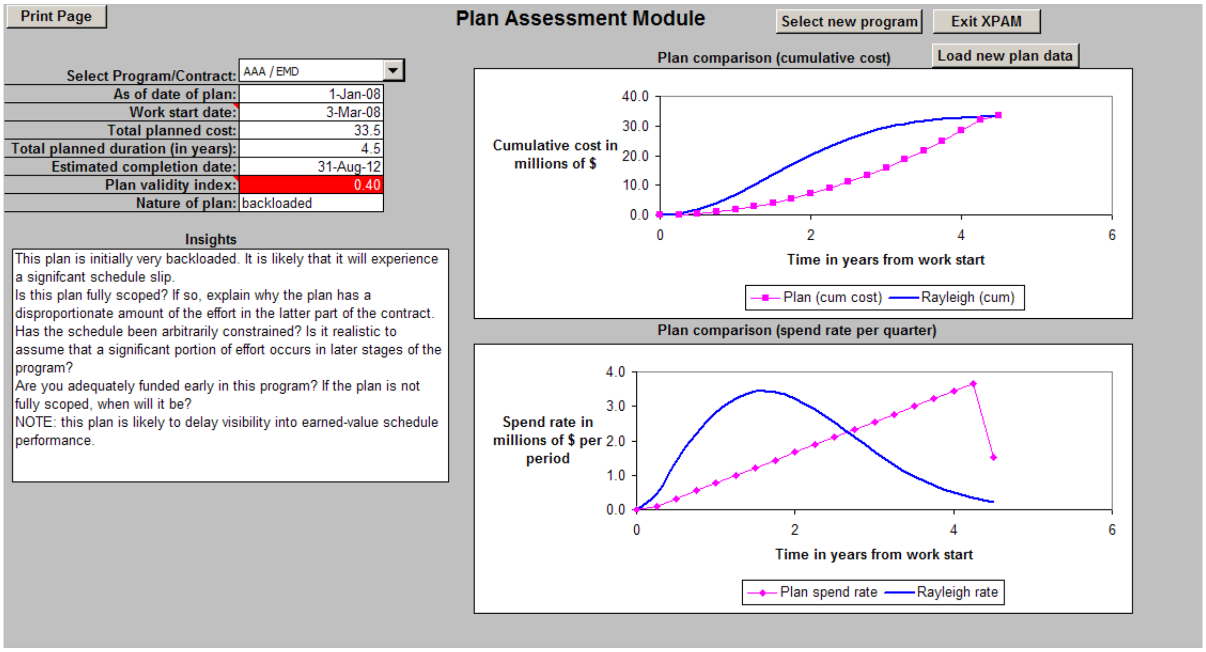


Select Program/Contract:
As of date of plan:
Work start date:
Total planned cost:
Total planned duration (in years):
Estimated completion date:
Plan validity index:
Nature of plan:

Insights

Press the "Select Program/Contract" drop-down and choose the desired plan to assess. Figure 16, below, shows the completed view of the plan assessment module.

Figure 30. Completed view of plan assessment module



Select Program/Contract: AAA / EMD
As of date of plan: 1-Jan-08
Work start date: 3-Mar-08
Total planned cost: 33.5
Total planned duration (in years): 4.5
Estimated completion date: 31-Aug-12
Plan validity index: 0.40
Nature of plan: backloaded

Insights

This plan is initially very backloaded. It is likely that it will experience a signifcant schedule slip.
Is this plan fully scoped? If so, explain why the plan has a disproportionate amount of the effort in the latter part of the contract.
Has the schedule been arbitrarily constrained? Is it realistic to assume that a significant portion of effort occurs in later stages of the program?
Are you adequately funded early in this program? If the plan is not fully scoped, when will it be?
NOTE: this plan is likely to delay visibility into earned-value schedule performance.

On the top right is a graph of the cumulative Rayleigh and the cumulative plan. On the lower right is a graph of the Rayleigh rate and plan rate of work accomplishment. The insights are keyed to the contract plan under consideration. The summary data appear in the upper left with a color-coded threshold for PVI.

To view a new plan, first press the "Select new program" button to clear existing data. Then select a new program/contract as described above.

If you wish to print a view, press the "Print page" button. When you are done, press the "Exit XPAM" button. This erases current data and returns you to the opening page. When you are done with the application, close the file and select "do not save."

Appendix D: Instructions for loading data

Loading data into XCAM

Open XCASA and enter the XCAM dashboard as described in appendix B. Select the "Load new data" button in the upper right section of the dashboard. This will take you to the location in the application where new contract data can be loaded.

All dates should be entered in the following day-month-year format: 01 Mar 1978.

Take care not to delete any rows or columns, as this will cause the code to operate incorrectly.

The required items must be entered, as a minimum.

Contract identification details

Required items

- Scroll to column BA.
- Enter the program name under column BA.
- Enter the program number (PNO) in column BB.
- Enter the contract number in column BC.
- Enter the contract name (CDES) in column BD.

Note that the program name and contract name should be the commonly accepted abbreviations. These names will be automatically merged and appear as a merged name in column AZ. This merged name along with the PNO and CNO are essential for selecting the correct data for the application to analyze. In addition, it is vital to enter the date that work under the contract started (WSDATE) in column BR. All dates should be entered in day-month-year format as in the examples included in the application (e.g. 01 May 1978).

Contract identification details should be entered with no gaps between rows of data. If a program's data are removed, ensure the remaining data are adjusted so there are no gaps.

Desired but optional data (to be entered if known)

- Enter the contract identification number in column BE.
- Leave column BF blank.
- Enter the contractor's city in Column BG.
- Enter the contractor's division in column BH.
- Enter the contractor's name in column BI.
- Enter the contractor's state in column BJ.
- Enter the contractor's ZIP Code in column BK.
- Enter the contract type (for example: FPAF for fixed price award fee) in column BL.
- Enter the contract definitization date in column BM.
- Leave column BN blank.
- Enter the word "Development" in column BO (remember only R&D contracts are amenable to analysis in XCASA).
- Enter the review date in column BP.
- Enter the source document for the data in column BQ (usually this will be CPR).
- Enter the review type in column BS.
- Enter the OSD CAIG estimate for EAC in column BT (if known).

Contract data

All contract data should be sorted vertically from earliest to latest "as of" date. The person entering the data should visually examine them to ensure that ACWP is never decreasing. That person should also ensure that there is only one entry of data for each "as of" date. If there is more than one entry with the same "as of" date, only the one that makes the most sense and that is not redundant should be kept. Eliminate all the rest.

Enter data for new contracts on lines numbered in multiples of 100. For example the first contract's data start on line 2. The second contract's data start on line 100, the third contract's data start on line 200, and so forth. This ensures data for different contracts do not overlap, and this is the way the application can quickly and accurately retrieve the correct contract data for analysis.

Required items

- Enter the PNO in column A and the CNO in column B. This must be done for each data entry for each contract, as this is the way data are correctly retrieved by the implementing code for analysis.
- Enter the date that data were submitted in column C (this date is often later than the "as of" date that should be recorded in column AE).
- Enter ACWP in column D.
- Enter the estimated completion date in column T.
- Enter the "as of" date in column AE (labeled RPD). The "as of date" is the basis for all analysis.
- Enter the total allocated budget (TAB) in column AG.

Each one of the above elements represents the minimum data entry requirements for each new contract data entry. This is the minimum information necessary to calculate Rayleigh estimates and to use the XCAM application. There can be no gaps in this information, from earliest entry to last entry.

Desired but optional data (to be entered if known)

- Enter authorized unpriced work (AUWORK) in column E.
- Leave column F blank.
- Enter BCWP in column G. Enter BCWS in column H.
- Leave column I blank.
- Enter CBB in column J.
- Enter the relevant contract change number in column K.
- Leave columns L, M, N, O, P, and Q blank.
- Enter the contractor's estimate of cost at completion in column R.
- Enter the delivery quantity in column S.
- Enter the management reserve (MR) in column U.
- Enter the negotiated cost in column V.
- Leave columns W and X blank.
- Enter the current OTB date (if applicable) in column Y.
- Leave columns Z and AA blank.
- Enter the PM's best-case estimate of cost at completion in column AB.
- Enter the PM's most-likely-case estimate of cost at completion in column AC.
- Enter the PM's worst-case estimate of cost at completion in column AD.
- Enter the significant effort completion date in column AF.
- Enter the total quantity in column AH.
- Enter the total price in column AI.

When you have completed loading data into XCAM, press the "Return to XCAM Dashboard" button in the top left view of the spreadsheet. Then press the "Exit XCAM" button, close the XCASA file, and press "save changes" to preserve the changes you have made to the database.

Loading data into XPAM

Open XCASA and enter the XPAM module as described in appendix B. Select the "Load new plan data" button in the upper right of the XPAM view. This will take you to the location in the application where new contract data can be loaded.

All dates should be entered in the following day-month-year format: 01 Mar 1978.

Take care not to delete any rows or columns, as this will cause the code to operate incorrectly.

The required items must be entered, as a minimum.

Contract details (all items required)

- Enter the common abbreviation for the program name in column L.
- Enter the common abbreviation for the contract name in column M.
- The application will automatically merge the two names and enter program/contract name in column K.
- Enter the PNO in column N.
- Enter the CNO in column O.
- Enter the plan period subdivision in column P (for example: monthly, quarterly, or annual; these are the only choices in the application's code).
- Enter the date when work under the contract started in column Q.

- Enter the estimated completion date in column R.
- Enter the “as of” date of the plan, if known.

Contract data (all items required)

Data should be sorted to enter the planning data from the first planning period to the last planning period. There should be no gaps in the data. Enter each contract’s dataset beginning on rows in multiples of 100 (for example the first contract’s data starts on line 2, the second contract’s data starts on line 100, the third contract’s data starts on line 200, and so forth). This ensures data for each contract are sufficiently separated so that the application can retrieve the correct data for analysis.

- Enter PNO in column A.
- Enter CNO in column B. The PNO and CNO have to be entered for each data element so the application can find and correctly retrieve the correct data for analysis.
- Enter the dollar value of work that is planned to be done in the first period, then the second period, and so on to the last period in column C.

When you have completed loading the data, press the “Return to plan assessment module” button located in the upper left view of the worksheet. Press the “Exit XPAM” button in the upper right portion of the view. Close the XCASA file and press “save changes” to preserve the changes you have made to XPAM’s database.

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Glossary

ACAT I	Acquisition category I
ACWP	Actual cost of work performed
BAC	Budget at completion
BCWP	Budgeted cost of work performed
BCWS	Budgeted cost of work scheduled
CAS	Contract analysis system
CBB	Contract budget base
CDES	Contract designation
CLIN	Contract line item number
CNO	Contract number
CPFF	Cost plus fixed fee
CPI	Cost performance index
DoD	Department of Defense
EAC	Estimated cost at completion
EVM	Earned value management
EVMS	Earned value management system
FFP	Firm fixed price
IBR	Integrated baseline review
MR	Management reserve
NU	Nonlinear least squares with unrestricted parameters

NRP	Nonlinear least squares with restricted parameters
OTB	Overtarget baseline
PM	Program manager
PNO	Program number
PVI	Plan validity indec
R&D	Research and development
RDT&E	Research, development, testing, and evaluation
SPI	Schedule performance index
TAB	Total allocated budget
VAC	Variance at completion
WSDATE	Work start date
XCAM	Executive contract analysis module
XCASA	Executive cost and schedule analysis tool
XPAM	Executive plan assessment module

List of figures

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