# Military Readiness, Budget Programming, and Economic Theory

Jim Jondrow



CPP-2014-U-007332-Final March 2014



Photo credit line:
US Navy image number 040603-N-5319A-011 Arabian Gulf (June 3, 2004) - USS George Washington (CVN 73), guided missile destroyer USS Bulkeley (DDG 84), cruiser USS Vella Gulf (CG 72), Military Sealift Command (MSC) Supply-class fast attack support ship USNS Supply (T-AOE 6) and Halifax-class frigate Her Majesty's Canadian Ship HMCS Toronto (FFH 333) participate in a strike group photo while operating in the Arabian Gulf. The Norfolk, Va. based aircraft carrier is on a scheduled deployment in support of Operation Iraqi Freedom (OIF). U.S. Navy photo by Photographer's Mate 1st Class Brien Aho (RELEASED)
The ideas expressed in this paper are those of the author. The paper does not represent the views of CNA, the Center for Naval Analyses, the Department of the Navy, or the Department of Defense.
Distribution unlimited

#### Copyright © 2014 CNA

The reproduction of this work for commercial purposes is strictly prohibited. Nongovernmental users may copy and distribute this document in any medium, either commercially or noncommercially, provided that this copyright notice is reproduced in all copies. Nongovernmental users may not use technical measures to obstruct or control the reading or further copying of the copies they make or distribute. Nongovernmental users may not accept compensation of any manner in exchange for copies. All other rights reserved.

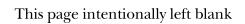
# **Contents**

Summary	1
Introduction	3
Budget programming and the production function	5
Terminology	6
The budget problem	11
An important ratio	11
The Constant Elasticity of Substitution Production Function.	13
Should budget be directed toward readiness deficiencies?	13
Can an important resource have a marginal product near 0?.	
A numerical example	15
More examples: a simple simulation	17
Concluding remarks	21
Conclusions	
Next steps	21
References	23
List of figures	25
List of tables	27

### **Summary**

This paper describes how readiness and performance metrics can inform budget programming. The paper draws on economic theory, using the production function, which connects military resources to military output (capability). It is assumed that the full production function is not known but some information is known, particularly the impact of small changes in each resource. A key to improved budgeting is the ratio of a resource's marginal product to its price. An example suggests that this ratio helps a budget move sharply toward the (unknown to programmers) optimum. A simple simulation suggests that knowing the entire production function would create substantial further improvement, but would require investment in better data. The paper considers whether directing funds toward readiness deficiencies is a good policy. In some cases it is not (where readiness is hard to produce or where a resource does not contribute greatly to output). The paper also considers how an important resource can have a near-0 marginal product. One case is when there is a lot of the resource relative to other resources.

<sup>&</sup>lt;sup>1</sup> I would like to thank Carlton Hill, Ron Nickel, and Sawyie Wang for valuable comments.

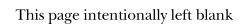


#### Introduction

Budget programmers in the military services are responsible for programming the budget for five future years, starting about two years in the future. All the military services do budget programming with the participation of other parts of DoD. Programming is the stage in budgeting where the first detailed budgets are put together—i.e., budget numbers are attached to programs. In addition to budget programming, the services and DoD collect a lot of data on current readiness and some data on performance. This paper discusses the following question: How can all these readiness and performance data be used to inform the programming process?

The paper answers the question using the economic concept of the production function, which expresses the relation between inputs and output. The idea of using the production function to analyze the readiness of military resources is not new (see [1, 2, 3, 4]). The paper is not meant to develop new theory in either economics or military analysis. Instead, it is meant to show how the two fit together—how basic economic theory can help with budget programming, and how budget programming can provide real-life material for economic theory to illuminate. The paper focuses on these questions:

- 1. What data are needed to make good programming decisions? What data should budget programmers be asking for?
- 2. How should these data be used in budget programming?
- 3. What would be the value of better data?



# Budget programming and the production function

The approach in this paper is to view budget programming as an application of microeconomic theory. Microeconomic theory includes the theory of the consumer and the producer. The paper considers the budget programmer as a producer, operating under limited information. The paper focuses on an example with two inputs of production (also called factors), X and Y. Each input is a military resource: for example, X could represent personnel and Y could represent training. The quantity of each resource is measured by its readiness metric. For example, a common index for personnel on a ship is "fit."

Let's look at fit in more detail. Fit is described in [5, p. 10] using the following example. Suppose that a ship's authorized billets in a particular specialty include 10 enlisted personnel, 8 at the pay grade of E-7 and 2 at the pay grade E-4. There are currently 9 enlisted personnel on board in this specialty, 6 at pay grade E-7 and 3 at pay grade E-4. In this case fit is 80%; two of the E-4s count toward requirements but the third E-4 does not, because the remaining requirement is at pay grade E-7.

Fit involves not only personnel, but also their Naval Enlisted Classifications<sup>2</sup> (NECs). NECs identify a "skill, knowledge, aptitude, or qualification." Let's expand the above example of fit to recognize the importance of NECs. Suppose now that the requirement is for personnel with the NEC 0340. In addition, one of the E-4 billets requires a second NEC 0342. All personnel on-board have the NEC 0340.

<sup>&</sup>lt;sup>2</sup> "The Navy Enlisted Classification (NEC) system, of which the NEC coding system is a part, supplements the enlisted rating structure in identifying personnel on active or inactive duty and billets in manpower authorizations. NEC codes identify a non-rating wide skill, knowledge, aptitude, or qualification that must be documented to identify both people and billets for management purposes" [6, chapter 1, p.1]

There are now 11 NECs required of which 8 can be filled for a fit of 73%. The assumptions and calculations are summarized in table 1.

Table 1. Example of fit calculation

						On-
Number			Addi-	Number		board
of re-			tional	of per-		Counted
quired	Re-	Re-	re-	sonnel	On-	toward
person-	quired	quired	quired	on-	board	require-
nel	rating	NEC	NEC	board	NEC	ments
5	E-7	0340		3	0340	3
3	E-7	0340		3	0340	3
1	E-4	0340	0342	2	0340	1
1	E-4	0340		1	0340	1
11 NECs required			8 NECs counted toward re-			
			quirements			

#### **Terminology**

Resources combine to produce output. The output might be measured as performance in an exercise or in combat. For example, the performance measure in an air defense exercise might be the expected number of incoming aircraft and missile targets that a carrier strike group (see cover illustration) can shoot down (simulated).

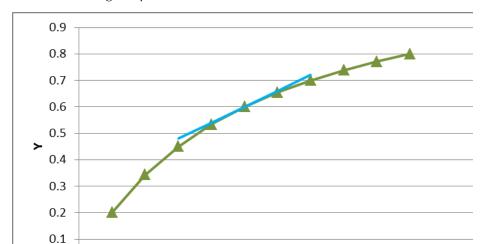
The function relating resources to output (Q), Q = f(X,Y), is called a *production function* in economics. There are also "prices" for X and Y ( $P_x$  and  $P_y$ ), called *input prices* in economics. Military resources X and Y may not have a simple price, because the costs of increasing readiness may not be proportional to the levels of the indexes. When proportionality is lacking,  $P_x$  and  $P_y$  are interpreted as the marginal costs of increasing the readiness indexes (e.g.,  $P_x$  would be the derivative of cost with respect to the quantity of X).

The correspondence of budgeting and economic terminology used in this paper is shown in table 2.

Table 2. Terminology from budget programming and from economics

Budget programming and readiness	Economic theory
Resources	Inputs or factors of production
Resource readiness	Input quantity
Performance or capability	Output
Derivative of cost with respect to readiness (which might be estimated in an analysis of resources to readiness)	Input price or factor price
Derivative of capability (or performance) with respect to readiness in a particular resource (which might be estimated in an analysis of resources to capability)	Marginal product

The limited information available to the budget programmer does not include the entire production function. My impression is that the current stage of knowledge is (at most) some information on the impact of various resources, where impact is measured at the current levels of the resources. These impacts are also called derivatives or "marginal products." The curve below shows a slice of the production function, the values of output for a given value of resource Y and varying values of resource X. The derivative or marginal product is the slope of the line just touching the curve.



0

0.2

Figure 1. A slice of the production function with line illustrating the marginal product

Studies of resources to readiness provide information on cost derivatives such as  $P_x$ . Studies of resources to performance begin to get at derivatives such as Q with respect to X. Information is also available from performance pricing models, which seek to quantify the consequences of different budgets.

0.4

0.6

Х

0.8

1

Why is full information on the production function not known? Why is getting valuable data so hard? Here are several possible reasons:

- Services will not intentionally or randomly choose an inefficient point.
- The range of variation in the data may be small.
- Data are expensive to construct. For example, performance data from exercises must be reconstructed and archived.
- The collection of data can run up against other interests. For example, the collection of performance data may be considered to conflict with the training function of the exercise.
- The individual resources have strong advocates.

• A lot of analytical questions in many fields are turning out to be more complex than originally thought, and the null hypothesis of no effect is proving hard to reject. Examples abound in the field of medicine. See, for example, [7] on the effect of mammography.

### The budget problem

I assume that the goal of budget programming is to maximize output for a given budget. This assumption requires some explanation. If you ask a budget programmer what the goal is, you may be told that the goal is to meet requirements. But, typically the budget is not large enough to meet all requirements, so there will be shortages or gaps. In this case, output is defined as some measure of the closeness to meeting requirements. Output can also be defined as the opposite of "programmatic risk," risk that can be as stark as a shortage of lifesaving protective gear. In the examples in this paper, output is measured as performance.

If the production function were known as well as factor prices, this maximization could be done exactly. In the absence of complete information about the production function, the goal is less ambitious, to answer the following question: Starting from some level of budget ("the Current Program of Record") and given the amount of budget available this year, what changes to resources, consistent with the budget, will result in the largest increase in output? Of course, this increase could be negative, depending on the change in the budget.

#### An important ratio

Given that complete information is not available on the production function, what use can be made of readiness metrics to inform budget programming? To answer this question, consider the effect of an increase in the budget to be allocated to one resource, to the other, or to both. Where should this increase be put in order to yield the greatest "bang for the buck?"

The change from putting the extra budget (dB) into X is approximately

$$dQ_x = f_x dX = (f_x/P_x)dB (1)$$

where  $f_x$  is the derivative of the function f with respect to x.

Similarly, the change from putting the extra budget into Y is

$$dQ_{y} = f_{y}dX = (f_{y}/P_{y})dB \tag{2}$$

 $dQ_x$  will exceed  $dQ_y$  if the following inequality is true; if so, budget increases should be directed toward X rather than Y, because directing the increases toward X will yield the larger increase in Q.

$$\frac{f_x}{P_x} > \frac{f_y}{P_y} \tag{3}$$

This ratio of the marginal product to the input price is described in [8, p. 64]: "The contribution to output of the last dollar expended upon each input must equal  $\mu$ . The multiplier  $\mu$  is the derivative of output with respect to cost with prices constant and quantities variable." The quotation refers to an optimal allocation of budget to X and Y, in which case the ratios for X and for Y will be equal. When the ratios are unequal, it is preferential to direct the added budget to the factor with the higher ratio. This ratio gives a clear meaning to the concept "bang for the buck."

The ratio has a special importance in budget programming. As noted above, programming starts from the "program of record"—which often appears to be last year's budget adjusted for some obvious changes such as new tasks or changes in force size. There's no guarantee that the starting point or ending point of the programming process achieves an optimum. Fortunately, inequality 3 points the way to budget changes that move toward an optimum, even if the optimum is never reached.

Inequality 3 helps identify the information that is important for making budget decisions. It is important to know the marginal product of a resource (the added output for an addition in input) and the factor price (the cost of added units of readiness in the resource). Note that

useful decisions can be made even if the whole production function is not known.

#### **The Constant Elasticity of Substitution Production Function**

For specificity, let's consider a particular production function, the Constant Elasticity of Substitution (CES) production function.

$$Q = A[bX^{-r} + (1-b)Y^{-r}]^{-1/r}$$
(4)

where A, b, and r are parameters<sup>3</sup> of the function. The elasticity of substitution [8, p. 62] is a measure of the degree to which the resources can be substituted for each other. A value of 0 means that the resources have to be used in fixed proportions (e.g., cars and tires). A very high value means that the resources can be readily substituted. The parameter r is equal to (1-s)/s, where s is the elasticity of substitution.

For the CES function, the ratio [8, p. 86] of marginal product to price for X takes the form

$$b \times \frac{(\frac{Q}{X})^{r+1}}{P_x \times A^r} \tag{5}$$

#### Should budget be directed toward readiness deficiencies?

This ratio from the CES production function sheds some light on a common idea about budgeting and readiness—that budget increases should go to resources with readiness deficiencies. If this idea is interpreted to mean that budget increases should go to the resources with the lowest value of X, we can see that the idea is not necessarily true.

For example, if  $P_x$  is high, the ratio may be low even though X is low. Intuitively, it may not be efficient to add money to a resource when output is expensive to produce from that resource—that is, when ei-

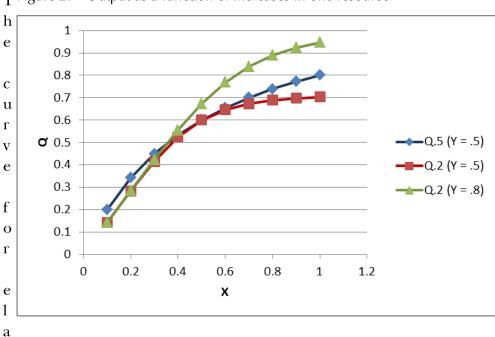
<sup>3.</sup> I'm using the term *parameter* to indicate a number that doesn't vary with X and Y.

ther  $P_x$  is high (readiness in resource X is expensive to produce) or b is low (resource X does not have a large effect on output).

#### Can an important resource have a marginal product near 0?

A special case of inequality 4 is when the empirical evidence does not suggest that the marginal product of a resource is greater than 0. How can this happen for an important resource? One possibility is that a low or 0 marginal product indicates that the quantity of the resource is high relative to other resources.

To see how this can happen, consider a CES function with X varying from 0.1 to 1, with Y set at .5 and coefficient A set at 1.2. If the elasticity of substitution is low (say, .2), the function becomes quite flat at high values of X. Figure 1 illustrates this case for elasticities of substitution of .5 and .2 and for two values of the other resource, resource Y.



T Figure 2. Output as a function of increases in one resource

sticity of substitution = .2 and Y = .5 becomes quite flat at high values for X. There is no "knee in the curve" where the curve flattens suddenly, but at high quantities of X it is unproductive to add more of resource X.

## A numerical example

Let's consider a numerical example to illustrate what can be done with limited information (derivatives) and what is the value of more information. We assume the following production function (repeated for convenience) and the following numerical values:

$$Q = A[bX^{-r} + (1-b)Y^{-r}]^{-1/r}$$
(6)

where

- Budget = 1.4
- $\bullet$   $P_{x}=2$
- $\bullet P_{v} = 1$
- Elasticity of substitution = s = .2, so r = (1-s)/s = .43
- Constant A = 1.2
- Share parameter b for X = .3
- X = .47
- Y = .46
- Q = .555.

How should a budget increase from 1.4 to 1.5 be allocated? The X ratio is .17 and the Y ratio is .87, indicating strongly that the budget increase should go disproportionately to Y. Suppose the budget increase is all dedicated to Y. The value of Q increases from .555 to .628. By comparison, if the budget increase were split evenly between X and Y, Q would increase to .606. Under perfect information, the optimal way of spending a budget of 1.5 would yield a Q of .636.

In this example, the limited information (the derivatives) allowed a pretty good decision to be made. A decision with full knowledge of the production function would increase output by .636 - .555 = .081. The decision (to dedicate the budget increase to Y) based on the derivatives increases output by about 90% of the full increase from moving to a full optimum with the complete production function. By

comparison, the "naïve" decision of splitting the budget increase evenly between X and Y increases output by about 60% of the full increase.

### More examples: a simple simulation

A small simulation generalizes the example in the previous section. The parameters were as shown above except that X was generated randomly with a uniform distribution running from .35 to .65. The simulation was run 100 times. For each run of the simulation, X was generated randomly, and then Y was calculated to satisfy the budget = 1.4. Then the budget was increased by .1 (7%) to 1.5. One strategy for using this increase is to split it evenly between X and Y. Another strategy is to spend the entire increase on the factor with the higher ratio of marginal product to price  $(f_x/P_x \text{ or } f_y/P_y)$ . A third strategy is to split the budget increase evenly unless the ratios are very different. For this third strategy, define  $R_x = (f_x/P_x)/(f_y/P_y)$  and  $R_y = (f_y/P_y)/(f_x/P_x)$ . If  $R = \max(R_x, R_y)$  exceeds some threshold, then spend the increase in budget on the resource with the higher ratio of marginal product to price. Otherwise, split the increase evenly between the two factors.

Results are shown in figure 2. (Points with value 5 and 10 on the horizontal axis are indicated with yellow fill.)

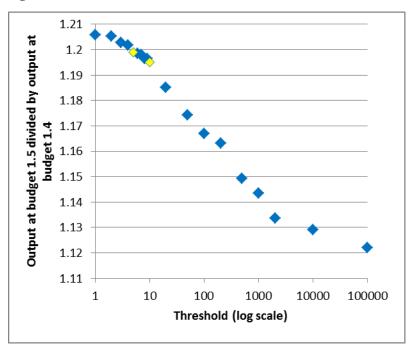


Figure 3. Simulation results

The horizontal axis is the threshold for R. A threshold of 1 means that the budget increase is always directed to the resource with the higher ratio of marginal product to price. Directing the budget increase to the resource with the higher ratio of marginal product to price raises output by about 21%, (from the starting point for output with budget = 1.4) a budget elasticity<sup>4</sup> of about 2.7 (since the budget increase was about 7%).

At a threshold of 100,000, the budget is almost always (always, in our sample of simulation runs) evenly split between the two resources. Splitting the budget increase evenly raises output by about 12%, an elasticity of about 1.7. A threshold of 2 works about as well as a threshold of 1, but larger thresholds don't work as well.

If the full production function were known, increasing the budget to 1.5 and redirecting this higher budget to the full optimum would increase output by about 42%, or about twice the increase from making the best decision shown on the chart. Thus, there would be a large

The elasticity of output with respect to budget is defined as  $d \ln(Q)/d \ln(B)$ , roughly the ratio of the percentage changes.

gain from the extra information necessary to achieve the full optimum.

Deriving the full production function would require substantial investment in more information. The scarcest information is data on performance in exercises and combat. The services and DoD should consider whether the value of these data would exceed the costs of reconstructing and archiving these data.

# **Concluding remarks**

#### **Conclusions**

- 1. Economic theory can help with budget programming.
- 2. Local derivatives of the production function can help with decisions, even if the entire production function is not known.
- 3. Economic theory includes a ratio that lends substance to the notion of "bang for the buck."
- 4. Knowing the full production function will add more value. A simulation suggested that this additional value would be substantial.
- 5. Readiness deficiency for a resource may not be a good reason for added budget for that resource.
- 6. An important resource can have a low marginal product if its quantity is large relative to other resources.

#### **Next steps**

This paper included limited numerical examples, where the derivatives proved an excellent guide to the best place to put extra budget dollars. It is important to consider a range of examples to see if cases are common where the derivatives are not a good guide.

Performance data are a clear case where the availability of the data falls far behind their importance. As Horowitz et al. noted in 1995 [9, p. S-3], "We need to devise procedures that will ensure that all of the Services produce, save, and assemble data on performance in combat." This need applies to exercise results as well.

#### References

- [1] Alan J. Marcus. *Personnel Substitution and Navy Aviation Readiness*. CNA Professional Paper 363. Oct. 1982.
- [2] Matthew T. Robinson et al. Avoiding a Hollow Force: An Examination of Navy Readiness. CNA Research Memorandum 95-238. Apr. 1996.
- [3] Omer E. Alper and Martha E. Koopman. *Fiscally Informed Manpower Requirements*. CRM D0014586.A2/Final. Sept. 2006.
- [4] Charles J. Hitch and Roland N. McKean. *The Economics of Defense in the Nuclear Age.* Cambridge: Harvard University Press. 1960.
- [5] Darlene E. Stafford and Michael J. Moskowitz. Eighth Annual Navy Workforce Research and Analysis Conference. Leading the Change: The Research Community in Navy's Strategic Vanguard. CNA Research Memorandum D0019587.A2/Final. January 2009.
- [6] Volume II: Navy Enlisted Classifications (NECs). U.S. Navy Bureau of Personnel, NAVPERS 18068F. January 2014.
- [7] Christie Aschwanden. "Mammography saves lives? Maybe it's not that simple." *Washington Post*, Tuesday Mar. 18, 2014, p. E4.
- [8] James M. Henderson and Richard E. Quandt. *Microeconomic Theory: A Mathematical Approach.* 2d ed. New York: McGraw-Hill, 1971.
- [9] Stanley Horowitz et al. *Unit Training in the Gulf War.* Institute for Defense Analysis document H 95-46822, IDA paper P-3087. Dec. 1995.

# List of figures

Figure 1.	A slice of the production function with line illustrating the marginal product	8
Figure 2.	Output as a function of increases in one resource	.14
Figure 3.	Simulation results	.18

# List of tables

Table 1.	Example of fit calculation	6
Table 2.	Terminology from budget programming and from	
	economics	7

#### CPP-2014-U-007332-Final

