A Model to Study: Cannibalization, FMC, and Customer Waiting Time

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Summary

The military services, the Department of Defense, and the U.S. Congress have all expressed concern about the shortages of spare parts for aviation units and about the workarounds, including the cannibalization of parts, that are required to achieve readiness goals. In this paper, we provide a theoretical framework that should help decision-makers understand why cannibalizations occur; what factors influence cannibalization rates; and, given the interaction of those factors, how to predict cannibalization rates.

Cannibalization has been defined as “the extent to which units of the armed forces remove serviceable parts, supplies, or equipment from one vehicle, vessel, or aircraft in order to render a different vehicle, vessel or aircraft operational [14].” Cannibalization is typically practiced when it is faster to remove a needed part from one aircraft and install it in another than to obtain that part from the supply system. Once the action is complete, the aircraft that received the part is rendered operational, and a new part is ordered to replace the one taken from the cannibalized aircraft.

Cannibalization often has a negative connotation. It is generally viewed as an indication that something is wrong with the supply system [15]. Some point to the fact that parts can be damaged during the process of cannibalization. Others say that cannibalization increases the workload of maintainers, and, if practiced too often, will reduce their morale [17].

1. See statements made in congressional hearings [1 through 5], two recent General Accounting Office (GAO) reports [6, 7], and articles in the media [8 through 13].

2. For recent empirical study on the cannibalization’s role in maintenance consolidation, see [15].
Ideally, cannibalization should not take place. If the allowance of aviation parts available on the ship (the AVCAL) is optimally designed, the system performs as expected, and the assumed reliability and expected supply response time from retail and wholesale supply are correctly specified, then the full-mission-capable (FMC) aircraft goal should be met without cannibalization. Cannibalization only becomes necessary in situations such as the following:

- The AVCAL is incorrectly constructed due to a funding shortfall or a shortage of parts.
- There is an unexpected demand for specific parts—for parts needed for aging aircraft, for example.\(^3\)
- A war or a local conflict creates an unexpected surge in the demand for parts.
- There is a change in system parameter(s)—in reliability, maintenance time, or customer wait times, for example.
- The maintenance crew needs to get an aircraft up “on the spot” to meet operational commitments.
- The FMC goal is raised.
- Maintainers use cannibalization as a diagnostic tool either because they lack the proper testing equipment or they lack the proper training to diagnose problems in a more efficient manner.
- The wait time for certain parts has increased because the manufacturer has cut back on production or because the vendor base is declining.

This paper presents a theory of cannibalization and a way to predict the cannibalization rates that are necessary to achieve a specified readiness goal, given expected customer wait times for the delivery of spare parts. We start with a description of the theoretical model and

\(^3\) Aging aircraft was cited as one of the causes of increasing cannibalization activities by GAO [6] and by Heimgartner and Zettler [3, 4]. Also see a recent study by Jondrow et al. [18].
then provide a numerical example. Next, we examine several policy implications and offer some suggestions for future research. Upon request, we will provide a spreadsheet calculator that will allow users in the Navy and Air Force to derive simulation results using their own parametric values.
The theoretical model and a numerical example

In this section, we will derive the relationship between cannibalization rates, customer waiting time (CWT) for needed spare parts, full-mission-capable rates (FMC), and gross effectiveness (GE).

We start with a number of assumptions. First, we assume that cannibalization affects Mean Supply Response Time (MSRT) only. Cannibalization is a maintenance activity, but its outcome is identical to that of a successful supply action. Maintenance time, which is measured by Mean Time To Repair (MTTR), is not affected by cannibalization so, for simplicity, we set the MTTR value as a constant. Cannibalization does not affect reliability as measured by the Mean Up Time (MUT)\textsuperscript{4} so we also assume that maintenance time and supply time are mutually exclusive. Hence, there are no overlapping maintenance and supply activities, i.e., the Mean Down Time (MDT) is the sum of MTTR and MSRT, or $\text{MDT} = \text{MTTR} + \text{MSRT}$.

We assume that every down incident requires exactly one part, and that supply time is zero when a part is available at the retail level. We recognize that some amount of time is required to satisfy a requisition, but, for simplicity, we have set this time at zero. When a part is not available at the retail level, the part must be ordered, and there is a certain amount of CWT before the part arrives. We assume that this CWT follows a continuous distribution over all parts that are not available at the retail level and that the distribution is stationary over time.

We assume that a fixed proportion of all aircraft are “down” for an extended period of time and that these aircraft are candidates for cannibalization. The down aircraft are not necessarily the same aircraft over time. Although the fact that the aircraft are down will obviously lower the FMC of the entire fleet, our model, for ease of

\textsuperscript{4} MUT is the expected uptime. For a detailed derivation and discussion of this definition and its relation to FMC, see appendix A.
exposition, computes only the FMC of the non-cannibalized aircraft. In footnote 7, we describe how the model can be adjusted to account for these down aircraft in order to calculate the FMC of the entire fleet.

Another assumption is that the required part is on the down aircraft and is available for cannibalization. We also assume that cannibalization activities take zero time to accomplish. Again, we know that some time is required to implement a cannibalization, but, for simplicity, we have set this time to zero.

Many of these assumptions can be relaxed to accommodate practical applications. In the last section, we will discuss some of these possible extensions of the model.

We start with the following definition of full mission capability, denoted FMC, for all aircraft:

$$FMC = \frac{MUT}{MUT + MDT}$$

We first derive the relationships between the FMC given by equation 1, the mean customer wait time for spare parts, denoted $\mu$, and the

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5. According to [19], not every part can be cannibalized. For example, in the F-16, about 25 percent of the parts (to include 44 items in the war reserve kit) are hard to cannibalize, and the same is true for about 19 of the parts (45 items) in the F-15. The A-10 has only ten or so items that are hard to cannibalize.

6. Here we model FMC rates only, but the model can be easily adapted to represent mission capable (MC) rates as well.

7. It is easy to adjust equation 1 for the “down” aircraft that are available to supply parts through cannibalization to the remaining aircraft in the fleet. Denote the proportion of cannibalized aircraft in the fleet as $q$. Then the FMC of the fleet, denoted as $FMC_{fleet}$ will be $FMC_{fleet} = (1 - q)$ * $FMC$, where $FMC_{fleet}$ is the FMC of the fleet, including the cannibalized aircraft, and FMC is the FMC of non-cannibalized aircraft given by equation 1. For simplicity, we omit such explicit treatment in this paper. Another version of this paper deals explicitly with this treatment and is available on request.
proportion of required parts that are available at the retail level, denoted as gross effectiveness (GE).

Without cannibalization, we have

2. $MSRT = (1 - GE)\mu$,

because gross effectiveness is defined as the proportion of required parts that are available at the retail level.

Thus, if the part is cannibalized immediately—because it takes zero time to decide to cannibalize and zero time to accomplish a cannibalization—we have

3. $MSRT = (1 - GE)(1 - c)\mu$,

where $c$ is the proportion of part requests that are not available at the retail level and are cannibalized. Note that without cannibalization, these cannibalized parts would have to be filled by an intermediate-level repair or an off-ship requisition.

Equation 3 also applies for an exponential distribution of customer waiting time if a rule requires that a maintainer must wait a certain amount of time, say $CWT^*$, before a spare part can be cannibalized. If the required part does not arrive within $CWT^*$ and the FMC goal is not being met, the maintainer will cannibalize the part. See appendix B for a derivation of this result.

The Navy usually describes cannibalization activity as the number of cannibalizations per 100 flight hours, termed the “cannibalization rate” and denoted as $CANN$. To get the conventionally defined cannibalization rate, $CANN$, and the proportion of all part requests that are cannibalized $c$, we let $\theta$ be the mean failure rate defined as the average number of failures per flight hour,

4. $c = CANN/\left[100(1 - GE)\theta\right]$.

If we plug equation 4 into equation 3, we have

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8. The Navy Aviation Management Program Manual [16] has set cannibalizations per 100 flight hours as a measurement criterion. See also [3,6].
5.  \( MSRT = [(1 - GE)\mu] - [CANN\mu/(100\theta)] \).

Note that equation 5 depicts a negative linear relationship between \( MSRT \) and \( CANN \). Hence, the higher the cannibalization rate, the lower is \( MSRT \), if we hold constant the failure rate, the expected customer wait time, and gross effectiveness.

Maintainers cannibalize in order to meet the FMC goal set for all aircraft in the squadron. If the FMC goal, denoted \( FMC_{goal} \), can be met for a given expected CWT, there is no need to cannibalize. If the goal can’t be met, cannibalization becomes necessary. If we plug equation 2 into equation 1 and solve for \( \mu \), we have

\[
6. \quad \mu (1 - GE) \leq \frac{MUT}{FMC_{goal}} - MUT - MTTR,
\]

and find that there is no need to cannibalize.

If equation 6 does not hold, cannibalization activities are needed to meet the FMC goal. The question is: How many cannibalization activities are needed in order to meet the FMC goal? To answer this question, plug equation 5 into equation 1 and solve for \( CANN \). We have

\[
7. \quad CANN = [100(1 - GE)\theta] - [100(MUT/FMC_{goal} - MUT - MTTR)\theta/\mu].
\]

Equation 7 depicts the tradeoffs between \( CANN \), \( FMC_{goal} \), and \( \mu \).

Equation 7 can be easily modified to accommodate the cannibalization definition used in the Air Force [4, 6]. Let \( CANN_{AF} \) be the number of cannibalizations per 100 sorties, and \( h_s \) the mean flight hours per sortie. We must have

\[
8. \quad CANN_{AF} = CANN * h_s.
\]

The model can also be adapted to accommodate alternative cannibalization measure(s) used by the other military services. 9

Finally, we need to impose two practical constraints on parameters when calculating the cannibalization rate with the formulas just

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9. The Army has three definitions for cannibalization type activities, but it has not yet defined a measurement. See [5, 6].
developed. First, because CANN is bounded below by zero, a constraint needs to be placed on the parameters in equation 7 to make them logically consistent. There is no cannibalization activity if expected CWT is very short, part availability at retail level (GE) is very high, and/or required FMC rate is very low. In other words, if the parameters set for the model imply \((1 - GE) \mu \leq \frac{MUT}{FMC} - MUT - MTTR\), as noted by equation 6, there is no need to cannibalize in order to meet the FMC goal. Mathematically, we have the following constraint:

\[9. \quad \text{CANN} = 0 \text{ if } (1 - GE) \mu \leq \frac{MUT}{FMC_{\text{goal}}} - MUT - MTTR.\]

For example, if \(GE = 0.75\), \(MTTR = 0.1\) days, and \(MUT = 1\) day, and if expected CWT is very short, say \(\mu = 2\), a squadron would not need to cannibalize to achieve an FMC goal of 0.625 or lower. Therefore, the model should constrain the CANN rate to 0 in this case.

Second, because MSRT is bounded by zero in the equation below, we need to place a constraint on the parameters in equation 1 in order for them to be logically consistent. Given MUT and MTTR, the maximum FMC, denoted \(FMC_{\text{max}}\), is

\[10. \quad FMC_{\text{max}} < \frac{MUT}{(MUT + MTTR)}.\]

Therefore, if \(FMC_{\text{goal}} > \frac{MUT}{(MUT + MTTR)}\), then parameter values set for the model are not logically consistent. It would be impossible to achieve an FMC goal that is higher than \(\frac{MUT}{(MUT + MTTR)}\).

We now apply a set of given parametric values to the formulas developed in this section. Let \(GE\) be 0.75, the failure rate \(\theta = 0.5\) failures per flight hour, \(MTTR = 0.1\) days, and \(MUT = 1\) day because each aircraft on average flies two hours per day. If we plug all these parameter values into equation 7 and keep CANN, \(FMC_{\text{goal}}\), and \(\mu\) as variables, we have

\[11. \quad \text{CANN} = 12.5 + \frac{(55 - 50/FMC_{\text{goal}})}{\mu}.\]

From equation 11, we have \(\text{CANN} = 6.8333\) when \(FMC_{\text{goal}} = 0.60\) and the expected CWT is \(\mu = 5\) days. The implied MSRT is 0.567 days. If we check these parameter values with equations 9 and 10, we find that all of them are logically consistent.
Moreover, from equation 11, as expected customer waiting time approaches infinity \((\mu \to \infty)\), and CANN reaches the maximum of 12.5 cannibalizations per 100 flight hours. Because \(GE\) is 0.75 and the failure rate is 0.5, a cannibalization rate of 12.5 cannibalizations per 100 flight hours means that maintainers cannibalize whenever a part is not available at the retail level.

Figure 1. CANN per 100 flight hours as a function of (average CWT) for various FMC goals

Figure 1 shows how CANN is related to \(\mu\) for three different values of the FMC goal: \(FMC_{goal1} = .55\), \(FMC_{goal2} = .60\), and \(FMC_{goal3} = .65\).10

Finally, if the mean flight hours per sortie is 2, i.e., \(h_s = 2\), then according to equations 8 and 11, the following equation gives the cannibalization measures for the Air Force:

12. \(CANN_{AF} = 6.25 + (27.5 - 25.0/FMC_{goal})/\mu\).

10. The curve can be shifted to reflect changes in other parameters such as \(GE\), reliability, and maintenance, but we have not done that here.
Policy implications

Now that we have established the relationship between CANN, E(CWT), FMC\textsubscript{goal}, MUT, and MTTR, we can address several policy implications.

First, we can draw a few conclusions by performing comparative static analyses on equation 7. Given the change in CANN with respect to various parameters, and by holding all other parameters fixed, we have the following intuitive results. Because $\frac{\partial \text{CANN}}{\partial \mu} > 0$, i.e., the longer the expected customer waiting time, the higher the cannibalization rate, all else being the same, we may reduce cannibalization activities by reducing the expected customer waiting time. A more efficient logistic system will allow the Navy to achieve a lower expected customer waiting time through better logistic organization, and better use of supply chain management and other efficient business practices. Because $\frac{\partial \text{CANN}}{\partial \text{MUT}} < 0$, i.e., the higher the reliability or the longer the mean uptime, the lower the cannibalization rate, all else being the same, cannibalization activities can be reduced with more reliable and better designed equipment. Because $\frac{\partial \text{CANN}}{\partial \text{MTTR}} > 0$, i.e., the longer the repair time (MTTR), the higher the cannibalization rate, all else being the same, one may reduce cannibalization activities with better trained and more qualified maintainers and a more efficient maintenance operation system. Finally, because $\frac{\partial \text{CANN}}{\partial \text{GE}} < 0$, i.e., the higher the GE the lower the cannibalization rate, all else being the same, cannibalization activities can be reduced by increasing the availability of parts.

One interesting point is the negative relationship between the higher reliability as measured by mean uptime (MUT) and the lower cannibalization rate. Note that MUT is a given parameter determined by reliability; it cannot be affected by cannibalization. However, should this reliability parameter change, it would affect the cannibalization rate negatively for given values of the other parameters in equation 7.
The next policy-related question is: What would be the impact on \( FMC_{goal} \) if we set a policy on the maximum limit to cannibalization activities? To answer the question, invert equation 7 and solve for \( FMC_{goal} \) to obtain

\[
13. \quad FMC_{goal} = \frac{100\theta \cdot MUT}{\left[100\theta \mu(1 - GE) + 100(MUT + MTTR) - CANN\mu\right]}.\]

Equation 13 gives the FMC rate should one set a CANN “goal” or a maximum number of allowed cannibalization activities. From equation 13, we see that the lower the allowed cannibalization activities, the lower the achievable FMC rate. Figure 2 depicts the relationship between FMC and a CANN goal when \( \theta = 0.5, \mu = 5, GE = 0.75, MUT = 1, \) and \( MTTR = 0.1. \)

A final policy question is this: What happens to CANN and FMC if we different cannibalization rules? For example, one could set a policy as to the maximum amount of customer waiting time, denoted \( CWT^* \), that would be allowed before taking cannibalization action. In appendix B, we show that all that is required to achieve a given FMC goal is a specified number of cannibalizations per hundred flight hours. This result holds true no matter how the rules for various \( CWT^* \) are defined as long as the customer waiting time is exponentially distributed.
Suggestions for future work

More work needs to be done. First, we need to have better and more complete measures of cannibalization activity, including types of cannibalization and reasons for cannibalization. In a recent report [6], GAO claimed that about 50 percent of the Navy’s cannibalization activities were not being reported.

Second, the model presented in this paper assumes that cannibalization requires zero maintenance time. Because the work involved in cannibalization is, in fact, a type of maintenance work, cannibalization should have a positive effect on maintenance time, i.e., $MTTR = F(CANN)$, and $F > 0$. Thus, more cannibalization activities imply longer maintenance time even if, in practice, this longer maintenance time is recorded as longer supply time.

Third, the model in this paper assumes that total downtime can be uniquely classified as the sum of maintenance time and supply time. In practice, there are three exclusive categories of downtime: maintenance time, supply time, and overlapping Mean Maintenance and Supply Time (MMST). To accommodate these different categories, equation 1 can be re-specified as $FMC = \frac{MUT}{MUT + MTTR + MSRT + MMST}$ because cannibalization affects MTTR and MSRT, as well as MMST.

Fourth, it is well recognized that cannibalization serves a useful purpose in the operation and maintenance of complex, high-performance equipment. Often, it is a necessary, viable, and even cost-efficient tool. Cannibalization becomes a serious problem only when it is practiced too often or in the wrong situation. There seems to be a “natural” rate of cannibalization, and we should know approximately what that rate is and incorporate it into the AVCAL design and other related budgets.\(^{11}\)

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11. For an earlier study along this line, see [20].
Finally, the model presented in this paper can be empirically tested with actual data on cannibalization rates and other measures related to readiness.
Appendix A: Definition of FMC

In this appendix, we give a formal definition of the full mission capable (FMC) rate. We define FMC as

\[ FMC = \frac{U}{U + D}, \]  

where \( U \) is the uptime and \( D \) the downtime, and both are random variables.

Moreover, \( D \) is decomposed into two mutually exclusive activities, \( S \) and \( M \), the supply time and maintenance time. Both are random variables, expressed as

\[ D = M + S. \]

If we plug equation A.2 into A.1, we have

\[ FMC = \frac{U}{U + M + S}. \]

We define MUT, MTTR, and MSRT as

\[ MUT = E(U), \]

\[ MTTR = E(M), \]

\[ MSRT = E(S). \]

For simplicity, we use the same notation, FMC, for expected FMC defined as

\[ FMC = \frac{MUT}{MUT + MTTR + MSRT}. \]

In figure 3, we use simulations to illustrate how FMC is calculated in practice to be consistent with the definitions of equation A.1 and A.5. In the figure, we demonstrate the concept of FMC. Suppose there are ten aircraft throughout a “month,” and a month is 30.4 days. We assume both uptime (in blue) and downtime (in red) are
exponentially distributed. The uptimes and downtimes shown in the figure are generated from these two exponential distributions, one with a mean uptime of 5 days and another with a mean downtime of 2.5 days. Seven out of the ten aircraft start the month with up status, and three start with down status. All aircraft are assumed to be identical, and we assume that the flight pattern is spread evenly throughout the day.

In our example, the steady-state FMC in the month is two-thirds, or approximately 66.7 percent, as computed by equation 1 in the text or by equations A.3 and A.4 above, where $E(M) + E(S) = 2.5$. That is, $FMC = E(U)/(E(U) + E(D)) = 5/(5 + 2.5) = .667$. Note that the steady-state FMC is given by $E(U)/[E(U) + E(D)]$ and not by $E[U/(U + D)]$. The FMC can be observed by taking a snapshot at one moment in time and computing the percentage of all aircraft in the fleet that are in the “up” status when this snapshot is taken. The average FMC of these snapshots once a steady state is reached will be 66.7 percent. Alternatively, one can select a fixed period of time, say 30.4 days, and compute FMC as the total uptime of all aircraft divided by the total time of all aircraft. Here the “total time of all aircraft” would be 30.4 days times the number of aircraft in the fleet. In the above example, once a steady state is reached the average value of these FMC computations would be 66.7 percent.

1. More sophisticated simulations of the theoretical model developed in this paper are also available upon request.
Appendix B

Consider a rule that requires a maintainer to wait a threshold time (CWT*) before a spare part is cannibalized. If the part does not arrive by CWT* and the FMC goal is not being reached, the maintainer cannibalizes the part.

Assume that CWT follows the “memoryless” exponential distribution with mean $\mu$. We have for the exponential distribution:

(B.1) $E(CWT) = \mu$.

(B.2) $E(CWT|CWT > CWT*) = CWT* + \mu$.

Equation B.1 says that if we do not cannibalize, the expected Mean Supply Response Time (MSRT) is $\mu$. Equation B.2 says that if the part did not arrive by CWT*, the expected additional time to wait for its arrival is also the mean waiting time $\mu$. Hence, if a part is cannibalized at CWT*, the expected reduction in Mean Supply Response Time is $\mu$ for each cannibalization. This reduction does not depend on the value of CWT*.

Let $c$ be the proportion of requisitions resolved by use of a cannibalization. Because each cannibalization will, on average, reduce the MSRT by $\mu$, the MSRT with cannibalization is given by

(B.3) $MSRT_{cann} = \mu - c\mu = (1 - c)\mu$.

Equation B.3 states that the mean supply response time with cannibalization depends only on $c$ (which depends on the number of cannibalizations per 100 flight hours that are required to achieve the FMC goal) and the expected customer wait time. The MSRT with cannibalization is independent of the policy rule reflected in different values of CWT*. For example, if 6.83 cannibalizations per 100 flight hours are required, on average, to achieve a certain FMC goal, it does not matter if these 6.83 cannibalizations are performed immediately or after waiting a specified length of time. If 6.83 cannibalizations per 100 flight hours are performed, the FMC goal will, on average, be achieved.


References


[14] Title 10, United States Code, Section 117(c)


[16] Naval Aviation Maintenance Program Manual, OPNAV 4790.2G, Section 12.1.11


